Selected answers from assignment 7
Math 105 History of Mathematics
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21. Solve the equation $16x^2 + 192x - 1863.2 = 0$ using Qin Jiushao's procedure.

This algorithm will find a solution to a polynomial equation in one unknown one digit at a time. Begin by writing, in columns here, the coefficients of the various powers of x.

x^2	x^1	x^0
16	192	−1863.2

The first job is to guess how many digits the answer will have. Will it be in the hundreds, tens, or what? If x is 10, a quick inspection shows $1600 + 1920$ is bigger than 1863.2, so 10 is too big.

So the answer is less than 10. Now 1863/192 is bigger than 9, so you might try 9, but probably not since you frequently find you need to try a smaller first digit. So try 8.

	x^2	x^1	x^0
	16	192	−1863.2
8		128	2480.0
	16	310	616.8

But 616.8 is positive, so 8 was too big a guess. Let's try 6.

	x^2	x^1	x^0
	16	192	−1863.2
6		96	1728
	16	288	−135.2
6		96	
	16	384	

Now we get to solve the equation $16y^2 + 384y - 135.2 = 0$, where $x = 6 + y$. And we know that y is less than 10. The first digit of 384, namely 3, goes into the first two digits of 135.2, namely 13, about 4 times giving us a guess that the next digit might be a 4. But 384 is about 400, and 4 goes into 13 only 3 times, so probably the next digit will be 3. Let's try it.

	y^2	y^1	y^0
	16	384	−135.2
.3		4.8	116.64
	16	388.8	−18.56
.3		4.8	
	16	393.6	

Now we get to solve the equation $16z^2 + 393.6z18.56 = 0$, where $y = .3 + z$. Well, 4 (about the first digit of 383.6) goes into 18 about 4 times. Let's try 4.

	z^2	z^1	z^0
	16	393.6	−18.56
.04		.64	15.7696
	16	394.24	−2.7904
.04		.64	
	16	394.88	

So far, our answer is 6.34. We could continue in this manner to get digit after digit, or we could divide 394.88 into 2.7904 to approximate the next few digits. 395 goes into 2.7904 about 0.0070643 times. That gives us a final answer of 6.3470643. How close is that to a solution of $16x^2 + 192x - 1863.2 = 0$? A modern calculator gives 6.34706443. It's only off by 1 in the 8th digit. Pretty good.

26. Solve Problem I, 4, from the *Shushu jiuzhang*, which is equivalent to the system of simultaneous congruences

$$\begin{aligned} N &\equiv 0 \pmod 5 \\ N &\equiv 4 \pmod 9 \\ N &\equiv 6 \pmod 8 \\ N &\equiv 0 \pmod 7 \end{aligned}$$

There are many ways to find the solution. Here we'll follow Qin Jiushao's algorithm precisely.

First compute the product of all the moduli. $5 \cdot 9 \cdot 8 \cdot 7 = 2520$.

Next for each modulus, compute the product of the remaining moduli, then reduce it modulo the given modulus

$$\begin{array}{rcrcl} 9 \cdot 8 \cdot 7 &=& 504 &\equiv& 4 \pmod 5 \\ 5 \cdot 8 \cdot 7 &=& 280 &\equiv& 1 \pmod 9 \\ 5 \cdot 9 \cdot 7 &=& 315 &\equiv& 3 \pmod 8 \\ 5 \cdot 9 \cdot 8 &=& 360 &\equiv& 3 \pmod 7 \end{array}$$

Next, for each of those numbers found in the last step, find the reciprocal modulo the given modulus. These moduli are small enough so that we can search for the solution For the first, we need to solve the congruence $4x \equiv 1 \pmod 5$. $x = 4$ will work. For the second, solve $1x \equiv 1 \pmod 9$, so, of course $x = 1$. For the third, solve $3x \equiv 1 \pmod 8$. $x = 3$ works. For the fourth, solve $3x \equiv 1 \pmod 7$. $x = 5$ works.

Now we can find a solution N to the system of linear congruences. To get N sum four numbers, each being a product of three numbers. The first of the three is the constant in the congruence, the second is the product of the remaining moduli for that modulus, and the third is the number found in the preceding step.

$$0 \cdot 504 \cdot 4 + 4 \cdot 280 \cdot 1 + 6 \cdot 315 \cdot 3 + 0 \cdot 360 \cdot 3 = 6790$$

Finally we can reduce that modulo the product 2520 of the four moduli to get the smallest positive number, namely $N = 1750$.

14. A problem from Mahavira: If 3 peacocks cost 2 coins, 4 pigeons cost 3 coins, 5 swans cost 4 coins, and 6 sarasa birds cost 5 coins, and if you buy 72 birds for 56 coins, how many of each type of bird do you have?

There are many methods you could use to find a solution. Here's one of them.

Let w be the number of trios of peacocks, x be the number of groups of four pigeons, y be the number of groups of five of swans, and z be the number of groups of six sarasa birds. Then you get two equations to solve simultaneously:

$$3w + 4x + 5y + 6z = 72$$

$$2w + 3x + 4y + 5z = 56$$

Subtracting the second from the first, we can simplify the first, then subtracting that from the second will reduce the two equations to

$$w + x + y + z = 16$$

$$w + 2x + 3y + 4z = 40$$

We can eliminate w from the equation by subtracting the first from the second to get

$$x + 2y + 3z = 24$$

There are several solutions with w, x, y, and z positive integers. You could take $w = x = y = z = 4$, for example, and that gives the solution 12 peacocks, 16 pigeons, 20 swans, and 24 sarasa birds.

A little more analysis will lead you to all solutions.

Math 105 Home Page at
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