

Creating Abstract Data Types (ADTs)

CS 121: Data Structures

START RECORDING

Outline

- Attendance quiz
- Overview of creating abstract data types (ADTs)
- Point charges
- Turtle graphics
- Complex numbers

Attendance Quiz

Attendance Quiz: Using ADTs

- Scan the QR code, or find today's attendance quiz under the "Quizzes" tab on Canvas
- Password: to be announced in class


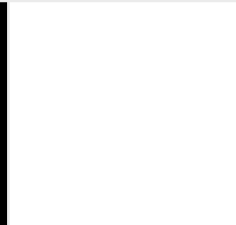
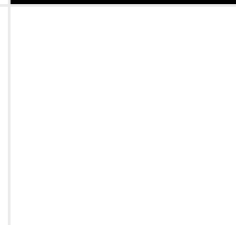
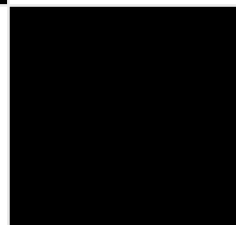


public class Color	
Color(int r, int g, int b)	
int getRed()	<i>red intensity</i>
int getGreen()	<i>green intensity</i>
int getBlue()	<i>blue intensity</i>
Color brighter()	<i>brighter version of this color</i>
Color darker()	<i>darker version of this color</i>
String toString()	<i>string representation of this color</i>
boolean equals(Color c)	<i>is this color the same as c's?</i>

public class Picture	
Picture(String filename)	<i>create a picture from a file</i>
Picture(int w, int h)	<i>create a blank w-by-h picture</i>
int width()	<i>width of the picture</i>
int height()	<i>height of the picture</i>
Color get(int col, int row)	<i>the color of pixel (col, row)</i>
void set(int col, int row, Color c)	<i>set the color of pixel (col, row) to c</i>
void show()	<i>display the image in a window</i>
void save(String filename)	<i>save the picture to a file</i>

Attendance Quiz: Using ADTs

- Write your name
- Using the Color and Picture ADTs below, implement a program Pic.java that will display this 2x2 pixel image:

<i>Index</i>	0	1
0		
1		

```
public class Color
```

```
    Color(int r, int g, int b)
```

```
    int getRed()           red intensity
```

```
    int getGreen()        green intensity
```

```
    int getBlue()         blue intensity
```

```
    Color brighter()      brighter version of this color
```

```
    Color darker()        darker version of this color
```

```
    String toString()     string representation of this color
```

```
    boolean equals(Color c) is this color the same as c's?
```

```
public class Picture
```

```
    Picture(String filename) create a picture from a file
```

```
    Picture(int w, int h) create a blank w-by-h picture
```

```
    int width() width of the picture
```

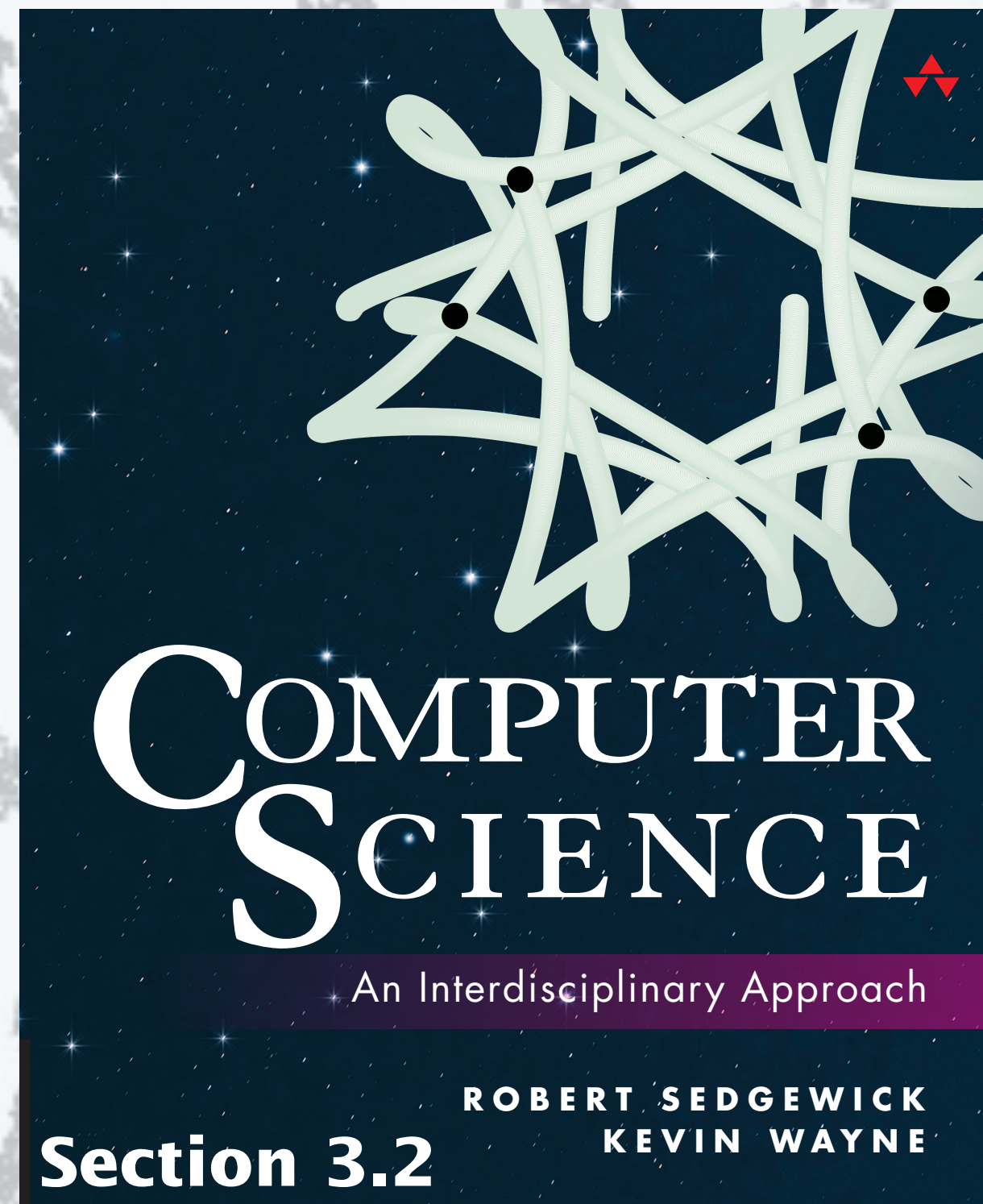
```
    int height() height of the picture
```

```
    Color get(int col, int row) the color of pixel (col, row)
```

```
    void set(int col, int row, Color c) set the color of pixel (col, row) to c
```

```
    void show() display the image in a window
```

```
    void save(String filename) save the picture to a file
```



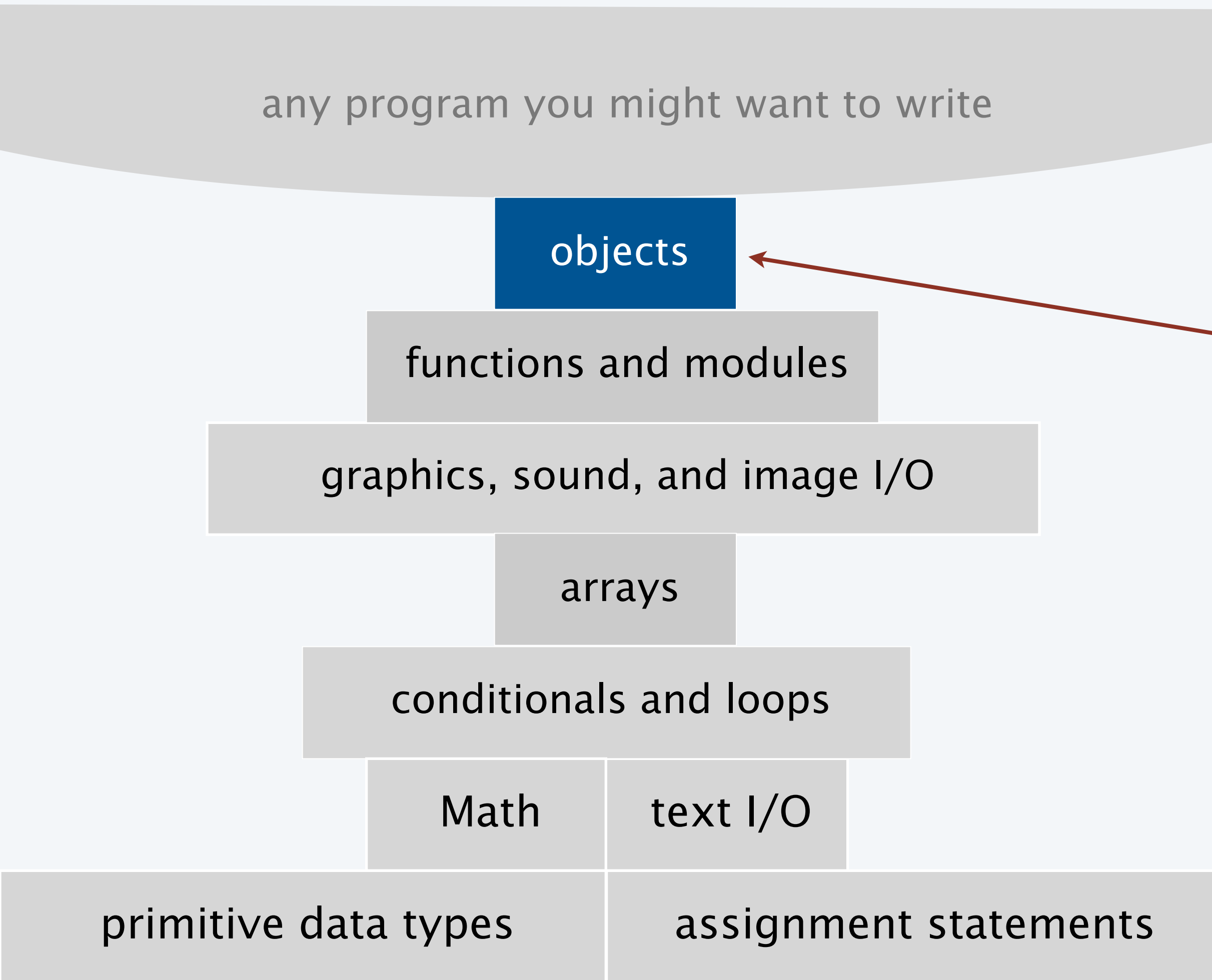
<http://introcs.cs.princeton.edu>

9. Creating Data Types

9. Creating Data Types

- **Overview**
- Point charges
- Turtle graphics
- Complex numbers

Basic building blocks for programming



Ability to bring life to your own abstractions

Object-oriented programming (OOP)

Object-oriented programming (OOP).

- Create your own data types.
- Use them in your programs (manipulate *objects*).

An **object** holds a data type value.
Variable names refer to objects.



Examples

<i>data type</i>	<i>set of values</i>	<i>examples of operations</i>
Color	three 8-bit integers	get red component, brighten
Picture	2D array of colors	get/set color of pixel
String	sequence of characters	length, substring, compare



An **abstract data type** is a data type whose representation is *hidden from the client*.

Impact: We can use ADTs without knowing implementation details.

- Previous lecture: how to write client programs for several useful ADTs
- This lecture: how to implement your own ADTs

Implementing a data type

To **create** a data type, your code must:

- Define the set of values (**instance variables**).
- Implement operations on those values (**methods**).
- Create and initialize new objects (**constructors**).

Instance variables

- Declarations associate variable names with types.
- Set of type values is "set of values".

Methods

- Like static methods.
- Can refer to instance variables.

Constructors

- Like a method with the same name as the type.
- No return type declaration.
- Invoked by `new`, returns object of the type.

In Java, a data-type implementation is known as a **class**.

A Java class



Anatomy of a Class

text file named
Charge.java

```
public class Charge
{
    private final double rx, ry; // position
    private final double q;     // charge value

    public Charge(double x0, double y0, double q0)
    {
        rx = x0;
        ry = y0;
        q = q0;
    }

    public double potentialAt(double x, double y)
    {
        double k = 8.99e09;
        double dx = x - rx;
        double dy = y - ry;
        return k * q / Math.sqrt(dx*dx + dy*dy);
    }

    public String toString()
    { return q + " at " + "(" + rx + ", " + ry + ")"; }

    public static void main(String[] args)
    {
        Charge c = new Charge(.72, .31, 21.3);
        StdOut.println(c);
        StdOut.printf("%6.2e\n", c.potentialAt(.42, .71));
    }
}
```

instance variables

constructor

methods

test client

not "static"

static method
(familiar)

```
% java Charge
21.3 at (0.72, 0.31)
3.61e+11
```



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PART I: PROGRAMMING IN JAVA

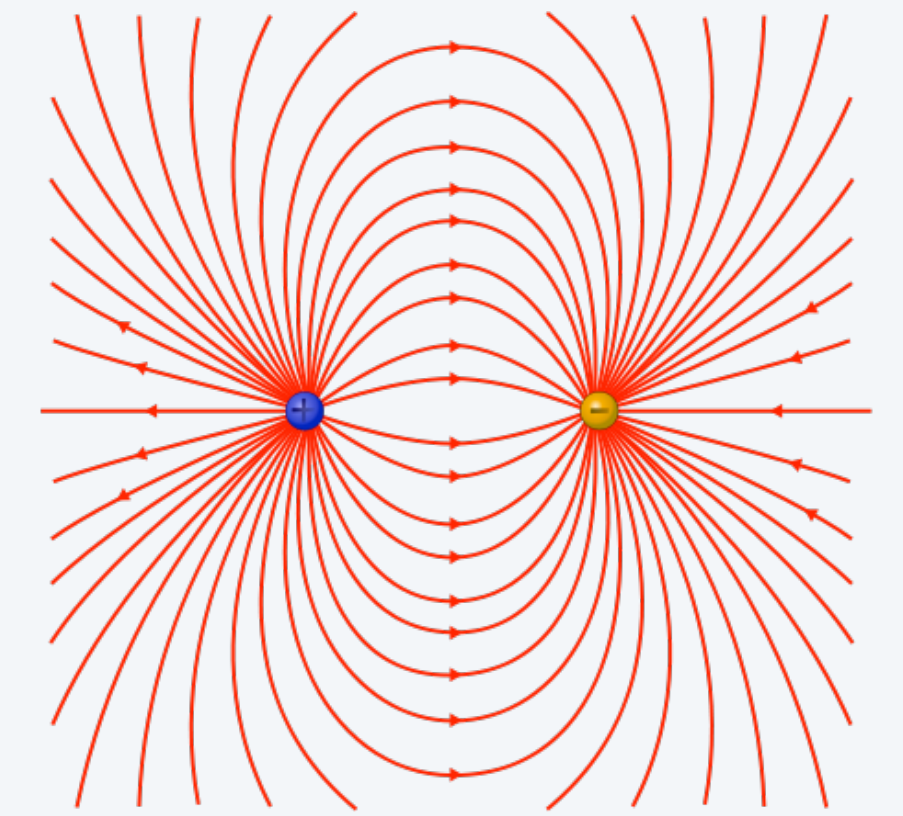
9. Creating Data Types

- Overview
- **Point charges**
- Turtle graphics
- Complex numbers

ADT for point charges

A **point charge** is an idealized model of a particle that has an electric charge.

An **ADT** allows us to write Java programs that manipulate point charges.



Values

	<i>examples</i>	
position (x, y)	(.53, .63)	(.13, .94)
electrical charge	20.1	81.9

API (operations)

```
public class Charge
```

<code>Charge(double x0, double y0, double q0)</code>	
<code>double potentialAt(double x, double y)</code>	<i>electric potential at (x, y) due to charge</i>
<code>String toString()</code>	<i>string representation of this charge</i>

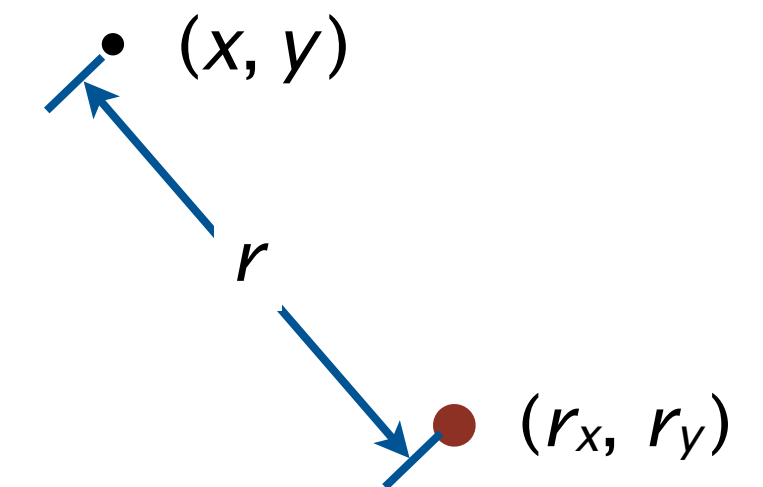
Crash course on electric potential

Electric potential is a measure of the effect of a point charge on its surroundings.

- It **increases** in proportion to the charge value.
- It **decreases** in proportion to the *inverse of the distance* from the charge (2D).

Mathematically,

- Suppose a point charge c is located at (r_x, r_y) and has charge q .
- Let r be the distance between (x, y) and (r_x, r_y)
- Let $V_c(x, y)$ be the potential at (x, y) due to c .
- Then $V_c(x, y) = k \frac{q}{r}$ where $k = 8.99 \times 10^9$ is a normalizing factor.



Q. What happens when multiple charges are present?

A. The potential at a point is the *sum* of the potentials due to the individual charges.

Note: Similar laws hold in many other situations.

← Example. *N*-body (3D) is an inverse *square* law.



Boston Museum of Science,
Theater of Electricity

Point charge implementation: Test client

Best practice. Begin by implementing a simple test client.

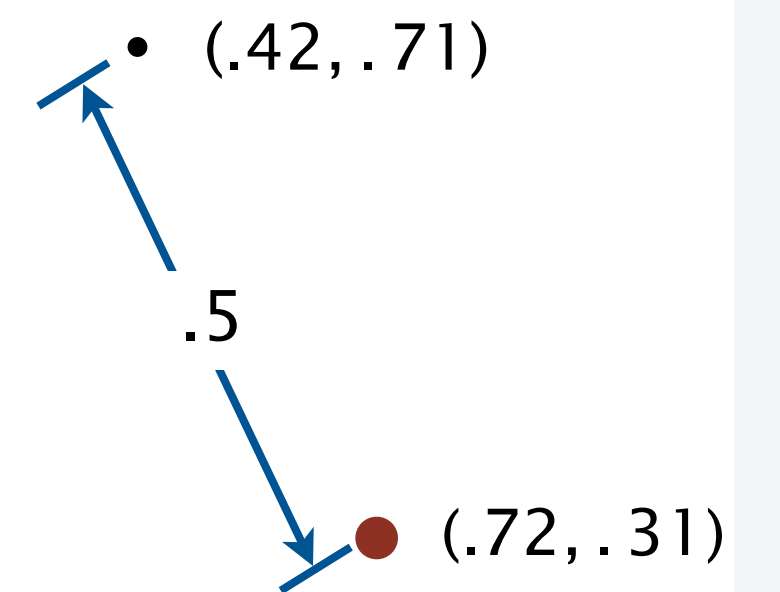
```
public static void main(String[] args)
{
    Charge c = new Charge(.72, .31, 20.1);
    StdOut.println(c); ← Reminder: automatically invokes c.toString()
    StdOut.printf("%6.2e\n", c.potentialAt(.42, .71));
}
```



$$V_c(x, y) = k \frac{q}{r}$$

$$r = \sqrt{(r_x - x)^2 + (r_y - y)^2}$$
$$= \sqrt{.3^2 + .4^2} = .5$$

$$V_c(.42, .71) = 8.99 \times 10^9 \frac{20.1}{.5}$$
$$= 3.6 \times 10^{11}$$



```
% java Charge
20.1 at (0.72, 0.31)
3.61e+11
```

← What we *expect*, once the implementation is done.

Point charge implementation: Instance variables

Instance variables define data-type values.

Values

	<i>examples</i>	
position (x, y)	(.53, .63)	(.13, .94)
electrical charge	20.1	81.9

```
public class Charge
{
    private final double rx, ry;
    private final double q;
    ...
}
```

Modifiers control access.

- **private** denies clients access and therefore makes data type abstract.
- **final** disallows any change in value and documents that data type is *immutable*.

↑
stay tuned

instance variables
constructors
methods
test client

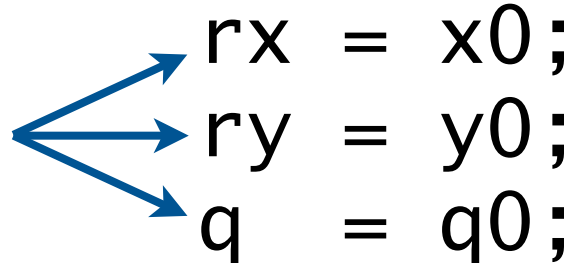
Key to OOP. Each *object* has instance-variable values.

Point charge implementation: Constructor

Constructors create and initialize new objects.

```
public class Charge
{
  ...
  public Charge(double x0, double y0, double q0)
  {
    rx = x0;
    ry = y0;
    q = q0;
  }
  ...
}
```

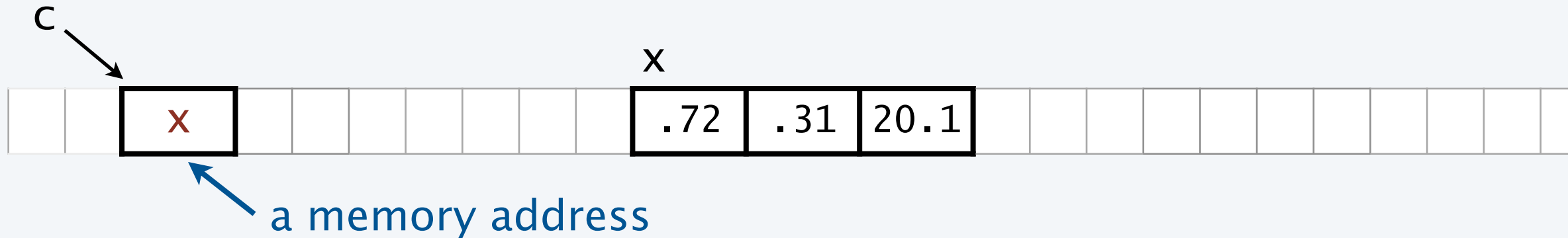
references to instance variables, which are *not* declared within the constructor



Clients use `new` to invoke constructors.

- Pass arguments as in a method call.
- Return value is reference to new object.

Possible memory representation of `Charge c = new Charge(.72, .31, 20.1);`



Point charge implementation: Methods

Methods define data-type operations (implement APIs).

API

```
public class Charge
```

```
    Charge(double x0, double y0, double q0)
```

```
    double potentialAt(double x, double y)
```

electric potential at (x, y) due to charge

```
    String toString()
```

string representation of this charge

```
public class Charge
```

```
{
```

```
    ...
```

```
    public double potentialAt(double x, double y)
```

```
    {
```

```
        double k = 8.99e09;
```

```
        double dx = x - rx;
```

```
        double dy = y - ry;
```

```
        return k * q / Math.sqrt(dx*dx + dy*dy);
```

```
    }
```

```
    public String toString()
```

```
    { return q + " at " + "(" + rx + ", " + ry + ")"; }
```

```
    ...
```

```
}
```

instance variables

constructors

methods

test client

Key to OOP. An instance variable reference in an instance method *refers to the value for the object that was used to invoke the method.*

Point charge implementation

text file named
Charge.java

```
public class Charge
{
    private final double rx, ry; // position
    private final double q; // charge value

    public Charge(double x0, double y0, double q0)
    {
        rx = x0;
        ry = y0;
        q = q0;
    }

    public double potentialAt(double x, double y)
    {
        double k = 8.99e09;
        double dx = x - rx;
        double dy = y - ry;
        return k * q / Math.sqrt(dx*dx + dy*dy);
    }
    public String toString()
    { return q + " at " + "(" + rx + ", " + ry + ")"; }

    public static void main(String[] args)
    {
        Charge c = new Charge(.72, .31, 20.1);
        StdOut.println(c);
        StdOut.printf("%6.2e\n", c.potentialAt(.42, .71));
    }
}
```

instance variables

constructor

methods

test client

```
% java Charge
20.1 at (0.72, 0.31)
3.61e+11
```

Point charge client: Potential visualization (helper methods)

Read point charges from StdIn.

- Uses Charge like any other type.
- Returns an array of Charges.

```
public static Charge[] readCharges()
{
    int N = StdIn.readInt();
    Charge[] a = new Charge[N];
    for (int i = 0; i < N; i++)
    {
        double x0 = StdIn.readDouble();
        double y0 = StdIn.readDouble();
        double q0 = StdIn.readDouble();
        a[i] = new Charge(x0, y0, q0);
    }
    return a;
}
```

Convert potential values to a color.

- Convert V to an 8-bit integer.
- Use grayscale.

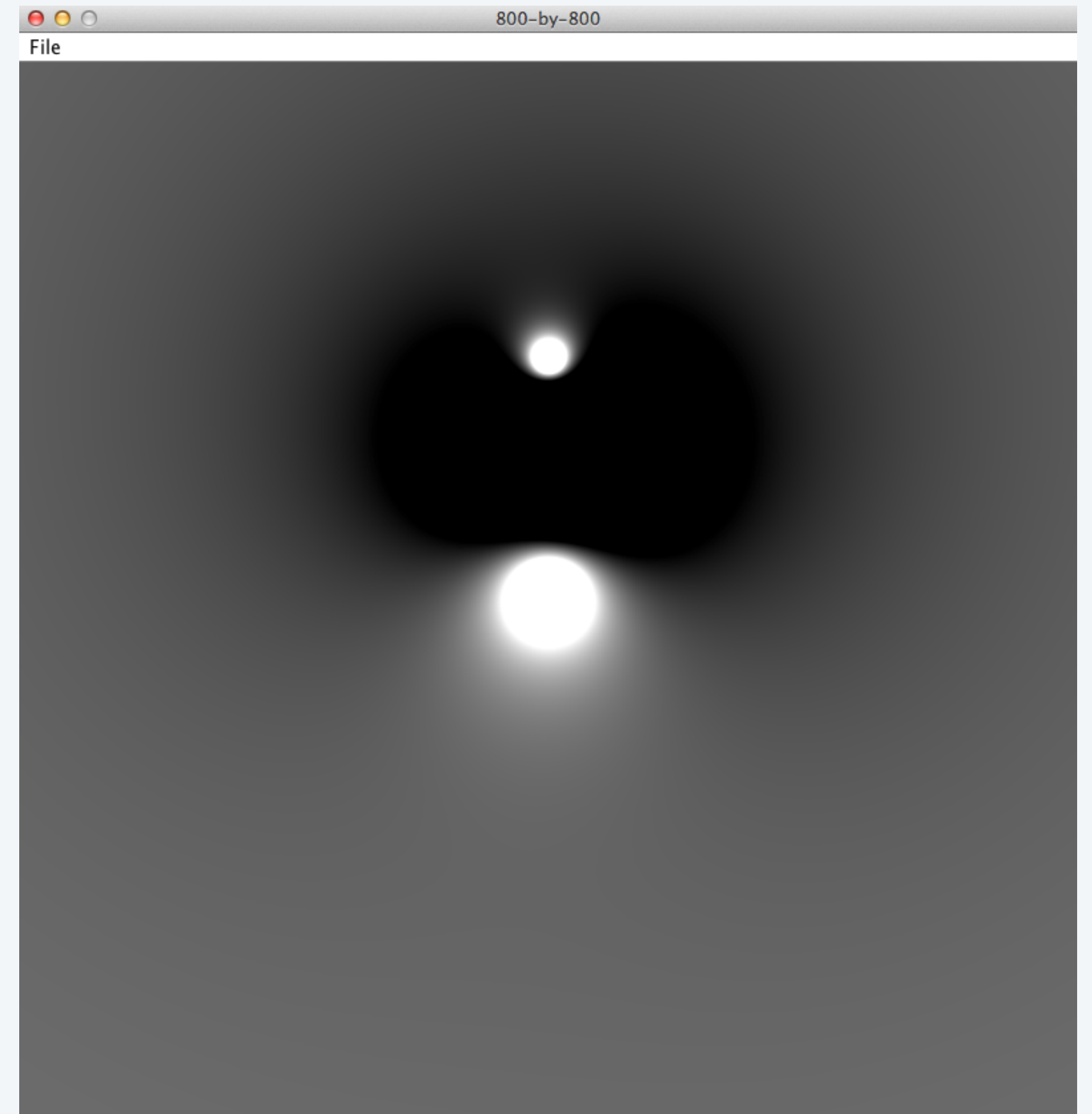
```
public static Color toColor(double V)
{
    V = 128 + V / 2.0e10;
    int t;
    if (V > 255) t = 255;
    else if (V >= 0) t = (int) V;
    else t = 0;
    return new Color(t, t, t);
}
```

V												
t	0	1	...	37	38	39	...	128	...	254	255	
	■	■		■	■	■		■				

Point charge client: Potential visualization

```
import java.awt.Color;
public class Potential
{
    public static Charge[] readCharges()
    { // See previous slide. }
    public static Color toColor()
    { // See previous slide. }
    public static void main(String[] args)
    {
        Charge[] a = readCharges();
        int SIZE = 800;
        Picture pic = new Picture(SIZE, SIZE);
        for (int col = 0; col < SIZE; col++)
            for (int row = 0; row < SIZE; row++)
            {
                double V = 0.0;
                for (int k = 0; k < a.length; k++)
                {
                    double x = 1.0 * col / SIZE;
                    double y = 1.0 * row / SIZE;
                    V += a[k].potentialAt(x, y);
                }
                pic.set(col, SIZE-1-row, toColor(V));
            }
        pic.show();
    }
}
```

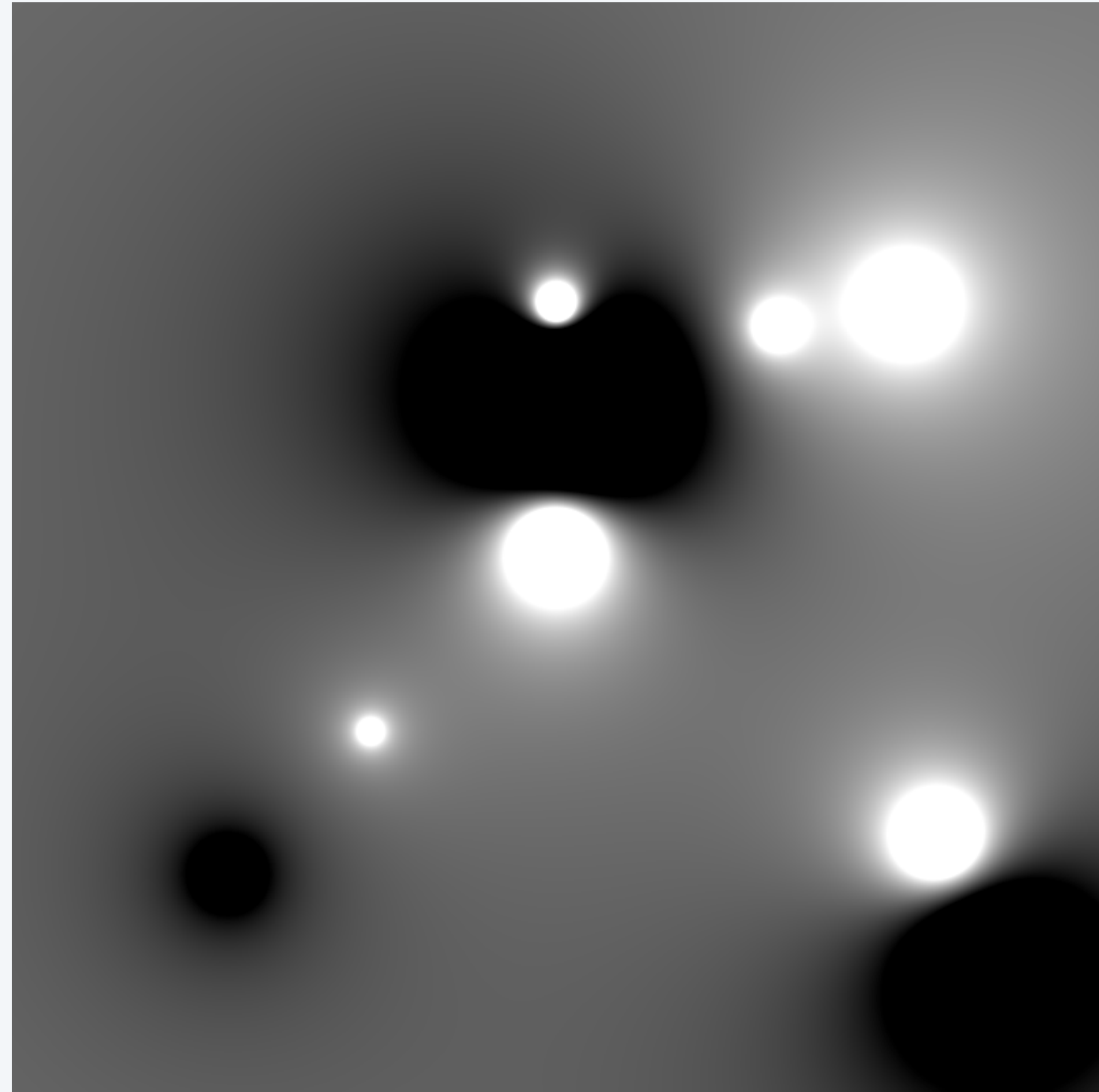
```
% more charges3.txt
3
.51 .63 -100
.50 .50  40
.50 .72  20
% java Potential < charges3.txt
```



Potential visualization I

```
% more charges9.txt
9
.51 .63 -100
.50 .50  40
.50 .72  20
.33 .33   5
.20 .20 -10
.70 .70  10
.82 .72  20
.85 .23  30
.90 .12 -50

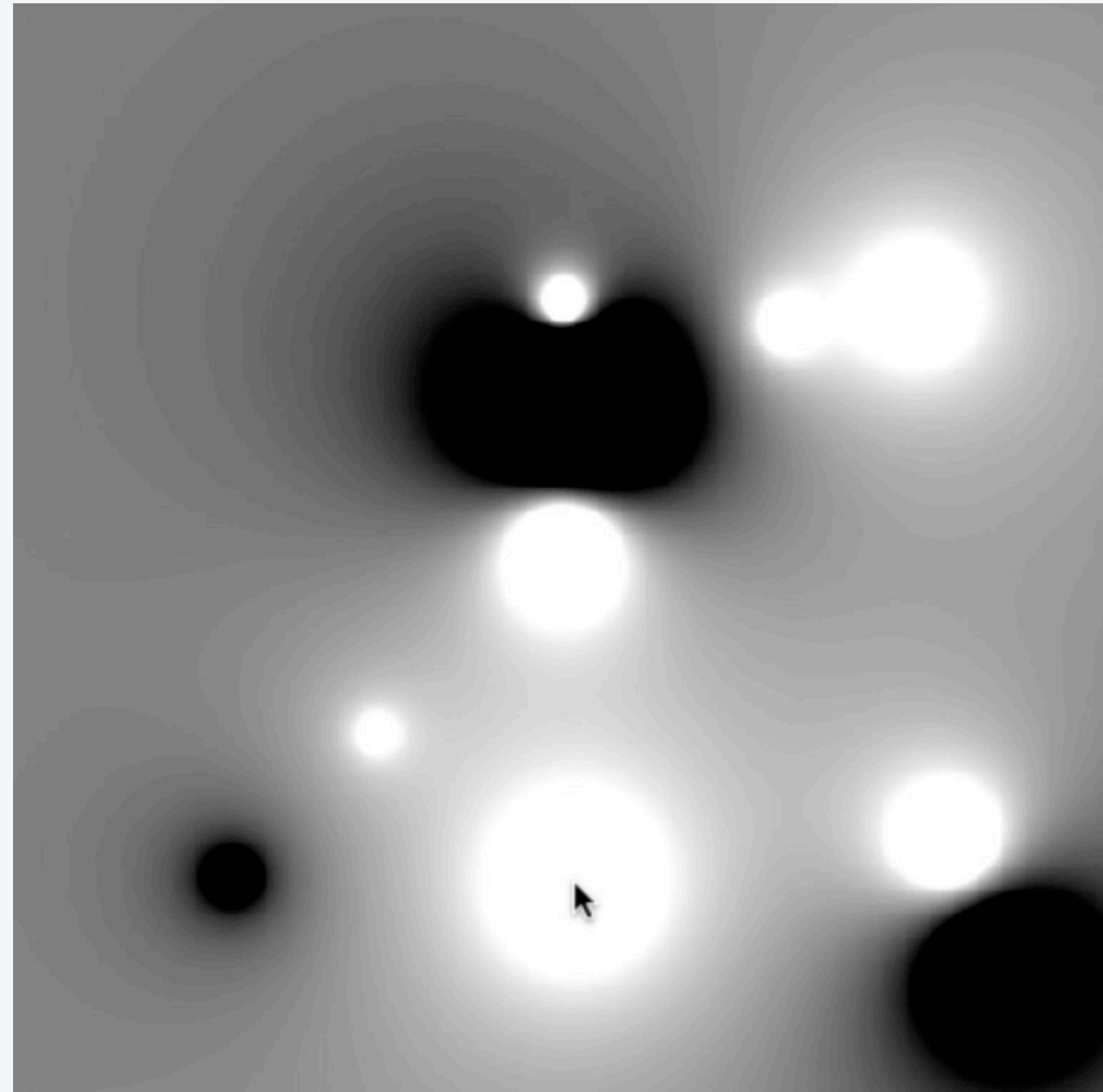
% java Potential < charges9.txt
```



Potential visualization II: A moving charge

```
% more charges9.txt
9
.51 .63 -100
.50 .50 40
.50 .72 20
.33 .33 5
.20 .20 -10
.70 .70 10
.82 .72 20
.85 .23 30
.90 .12 -50

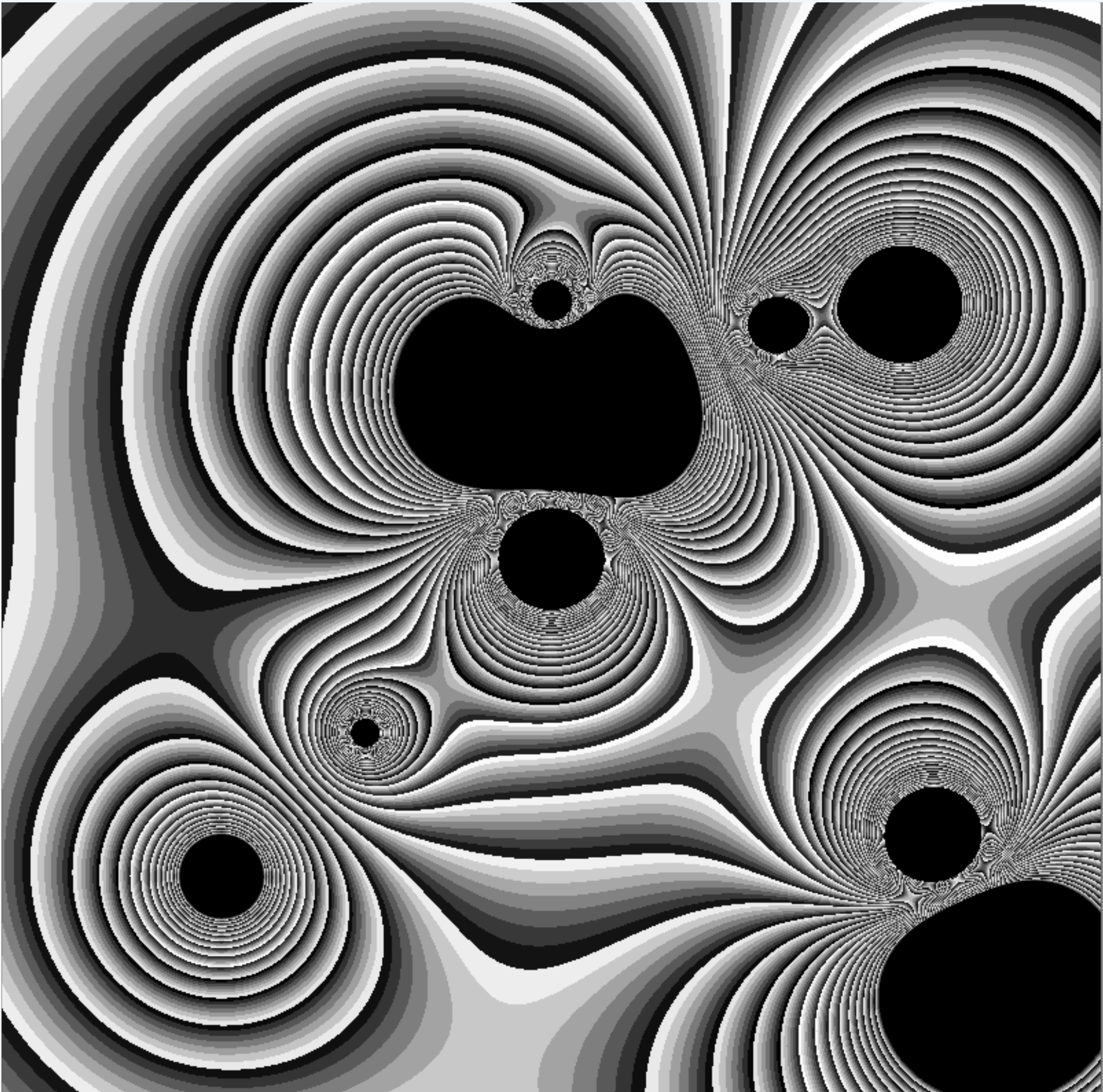
% java PotentialWithMovingCharge < charges9.txt
```



Potential visualization III: Discontinuous color map

```
public static Color toColor(double V)
{
    V = 128 + V / 2.0e10;
    int t = 0;
    if (V > 255) t = 255;
    else if (V >= 0) t = (int) V;
    t = t*37 % 255
    return new Color(t, t, t);
}
```

V											
t	0	1	2	3	4	5	6	7	8	9	...
	█	█	█	█	█	█	█	█	█	█	█





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PART I: PROGRAMMING IN JAVA

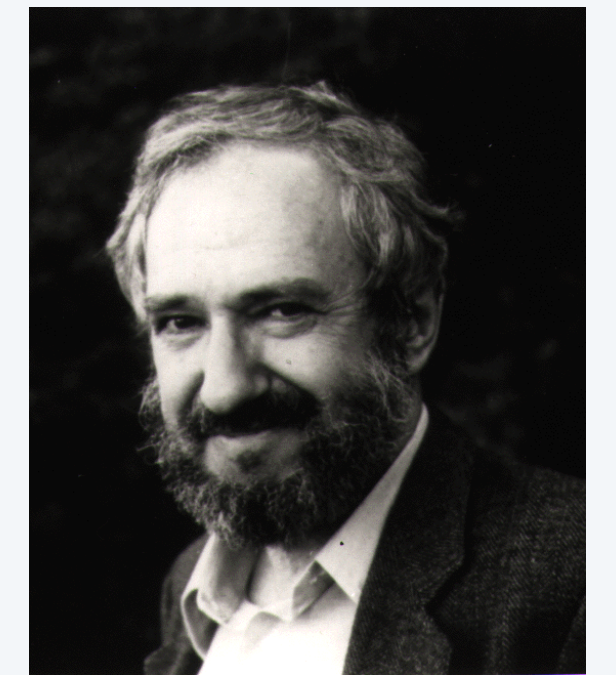
9. Creating Data Types

- Overview
- Point charges
- **Turtle graphics**
- Complex numbers

ADT for turtle graphics




A **turtle** is an idealized model of a plotting device.

An **ADT** allows us to write Java programs that manipulate turtles.



Seymour Papert
1928–2016

Values

position (x, y)	(.5, .5)	(.25, .75)	(.22, .12)
orientation	90°	135°	10°
			



API (operations)

```
public class Turtle
```

```
    Turtle(double x0, double y0, double q0)
```

```
    void turnLeft(double delta)
```

rotate delta degrees counterclockwise

```
    void goForward(double step)
```

move distance step, drawing a line

Turtle graphics implementation: Test client

Best practice. Begin by implementing a simple test client.

```
public static void main(String[] args)
{
    Turtle turtle = new Turtle(0.0, 0.0, 0.0);
    turtle.goForward(1.0);
    turtle.turnLeft(120.0);
    turtle.goForward(1.0);
    turtle.turnLeft(120.0);
    turtle.goForward(1.0);
    turtle.turnLeft(120.0);
}
```

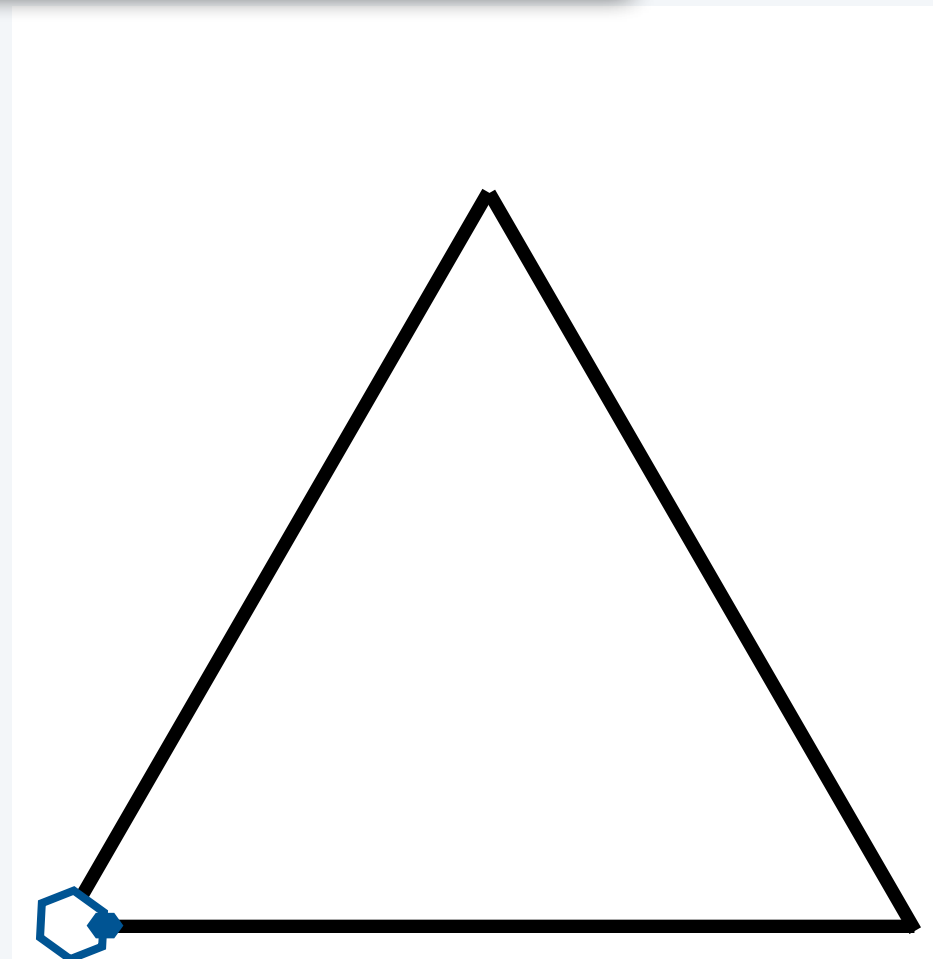
instance variables

constructors

methods

test client

% java Turtle



Note: Client drew triangle without computing $\sqrt{3}$

What we *expect*, once the implementation is done.

Turtle implementation: Instance variables and constructor

Instance variables define data-type values.

Constructors create and initialize new objects.

```
public class Turtle
{
    private double x, y;
    private double angle;
    public Turtle(double x0, double y0, double a0)
    {
        x = x0;
        y = y0;
        angle = a0;
    }
    ...
}
```

instance variables
are *not* final



	position (x, y)	(.5, .5)	(.75, .75)	(.22, .12)
	orientation	90°	135°	10°
Values				

Turtle implementation: Methods

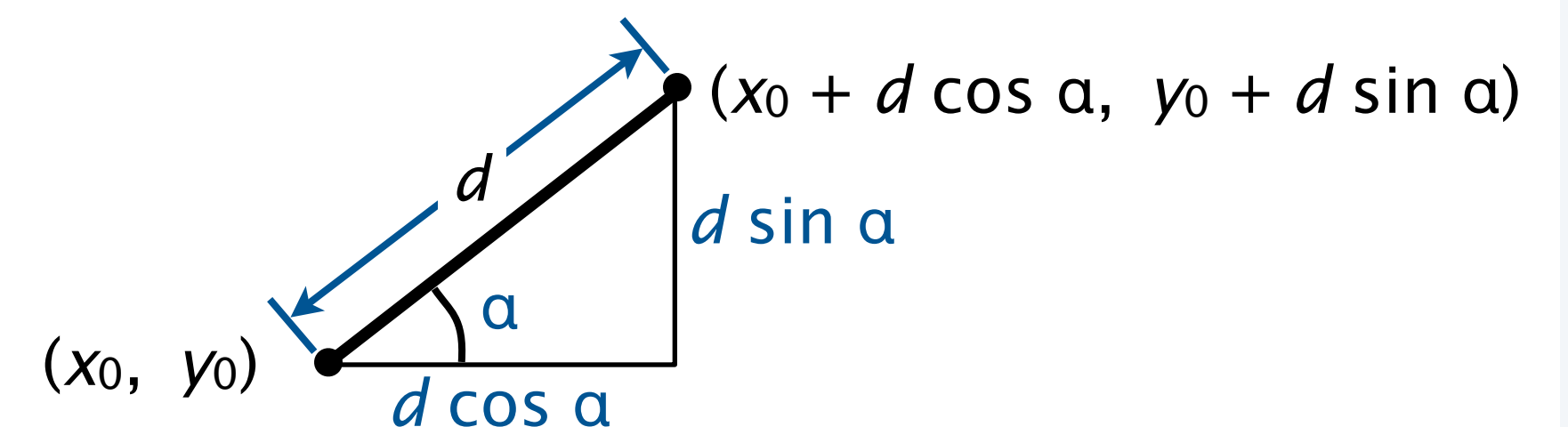
Methods define data-type operations (implement APIs).

```
public class Turtle
{
    ...
    public void turnLeft(double delta)
    { angle += delta; }
    public void goForward(double d)
    {
        double oldx = x;
        double oldy = y;
        x += d * Math.cos(Math.toRadians(angle));
        y += d * Math.sin(Math.toRadians(angle));
        StdDraw.line(oldx, oldy, x, y);
    }
    ...
}
```



API

```
public class Turtle
{
    Turtle(double x0, double y0, double q0)
    void turnLeft(double delta)
    void goForward(double step)
}
```



Turtle implementation

text file named
Turtle.java →

```
public class Turtle  
{
```

```
private double x, y;  
private double angle;
```

← instance variables

```
public Turtle(double x0, double y0, double a0)  
{  
    x = x0;  
    y = y0;  
    angle = a0;  
}
```

← constructor

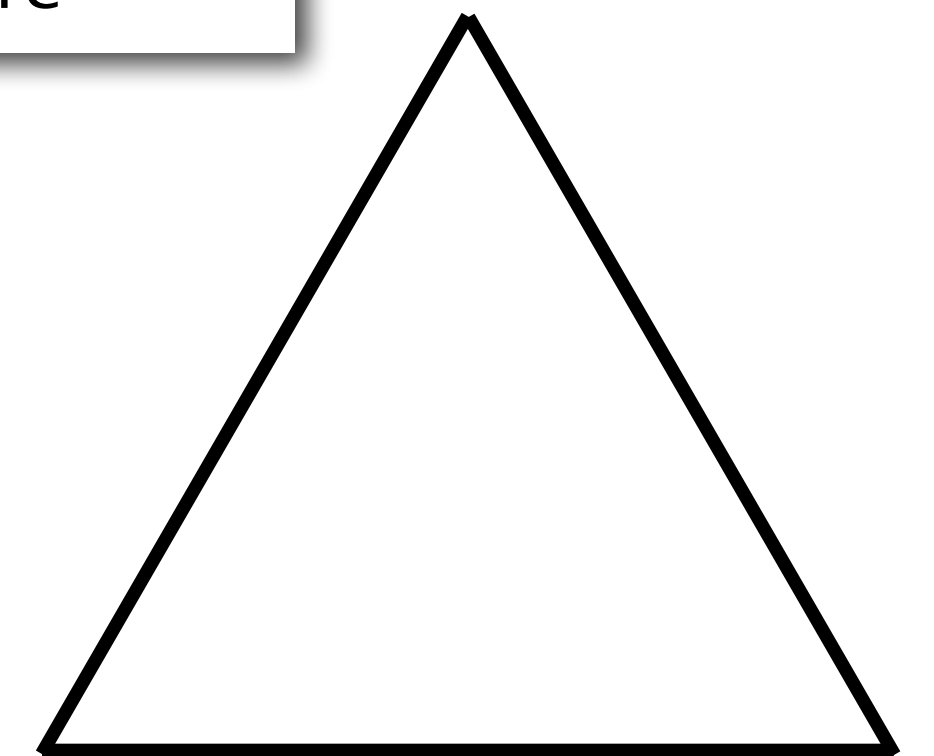
```
public void turnLeft(double delta)  
{ angle += delta; }  
public void goForward(double d)  
{  
    double oldx = x;  
    double oldy = y;  
    x += d * Math.cos(Math.toRadians(angle));  
    y += d * Math.sin(Math.toRadians(angle));  
    StdDraw.line(oldx, oldy, x, y);  
}
```

← methods

```
public static void main(String[] args)  
{  
    Turtle turtle = new Turtle(0.0, 0.0, 0.0);  
    turtle.goForward(1.0); turtle.turnLeft(120.0);  
    turtle.goForward(1.0); turtle.turnLeft(120.0);  
    turtle.goForward(1.0); turtle.turnLeft(120.0);  
}
```

← test client

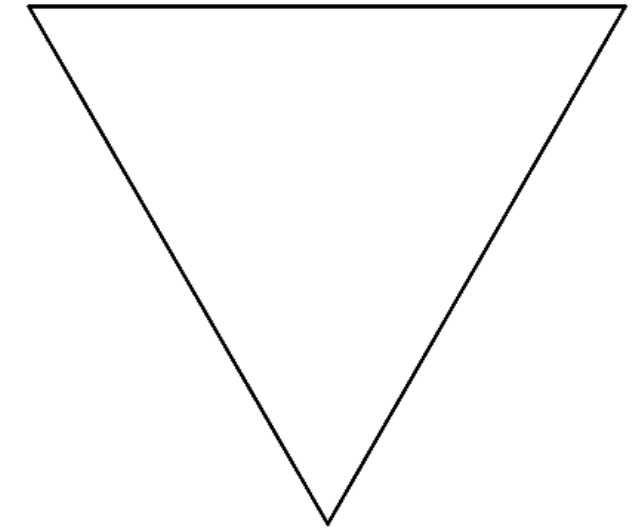
```
% java Turtle
```



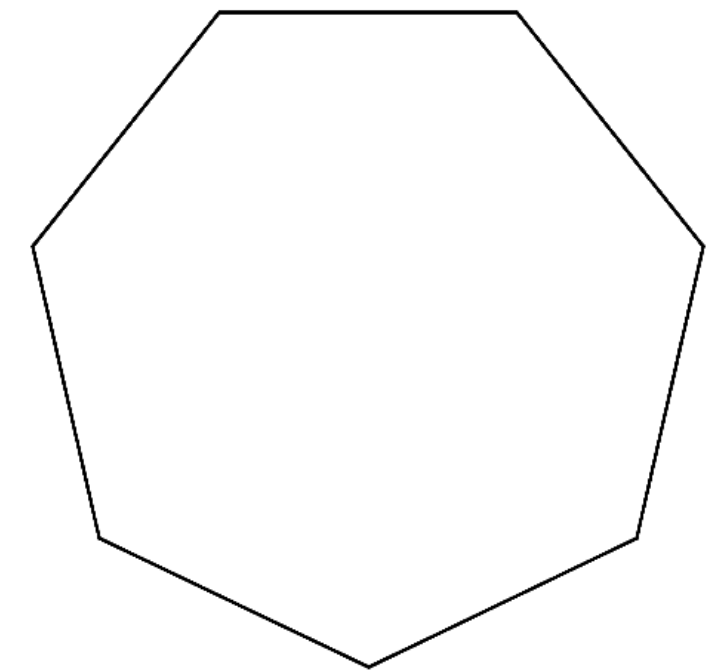
Turtle client: N-gon

```
public class Ngon
{
    public static void main(String[] args)
    {
        int N      = Integer.parseInt(args[0]);
        double angle = 360.0 / N;
        double step  = Math.sin(Math.toRadians(angle/2.0));
        Turtle turtle = new Turtle(0.5, 0, angle/2.0);
        for (int i = 0; i < N; i++)
        {
            turtle.goForward(step);
            turtle.turnLeft(angle);
        }
    }
}
```

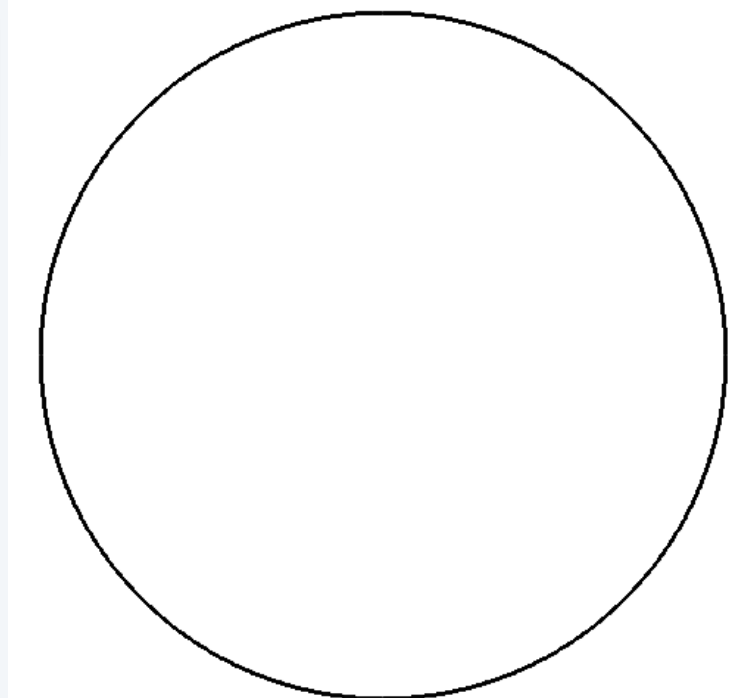
% java Ngon 3



% java Ngon 7



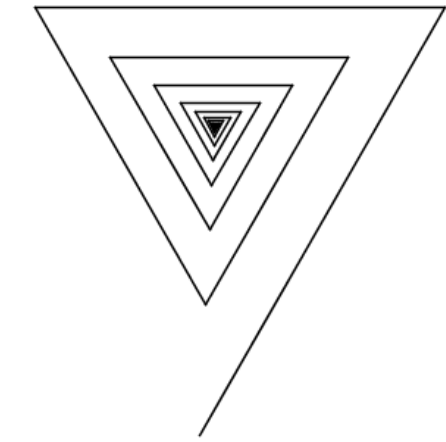
% java Ngon 1440



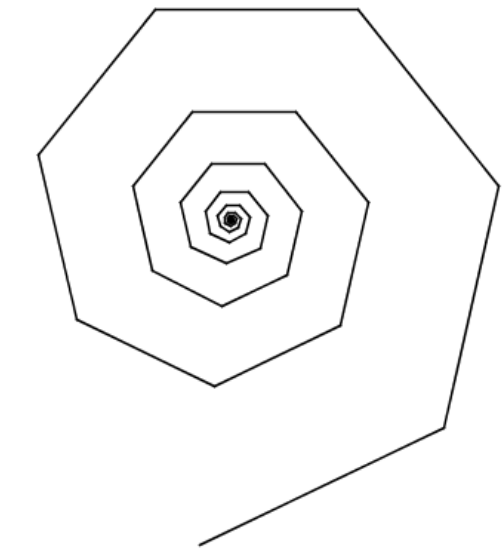
Turtle client: Spira Mirabilis

```
public class Spira1
{
    public static void main(String[] args)
    {
        int N      = Integer.parseInt(args[0]);
        double decay = Double.parseDouble(args[1]);
        double angle = 360.0 / N;
        double step  = Math.sin(Math.toRadians(angle/2.0));
        Turtle turtle = new Turtle(0.5, 0, angle/2.0);
        for (int i = 0; i < 10 * N; i++)
        {
            step /= decay;
            turtle.goForward(step);
            turtle.turnLeft(angle);
        }
    }
}
```

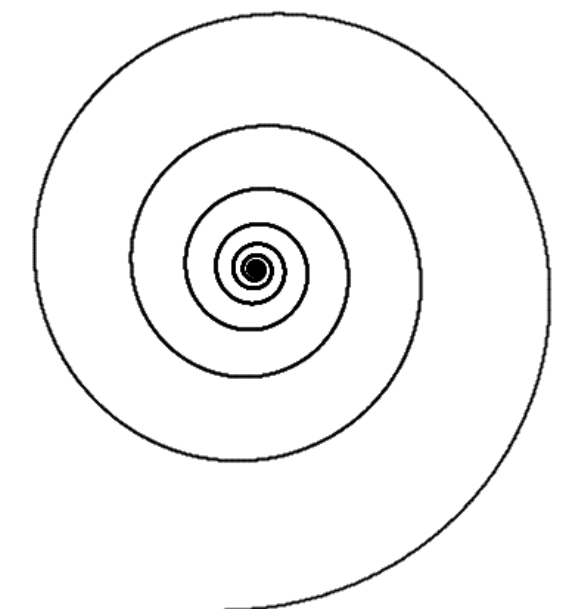
```
% java Spira1 3 1.2
```



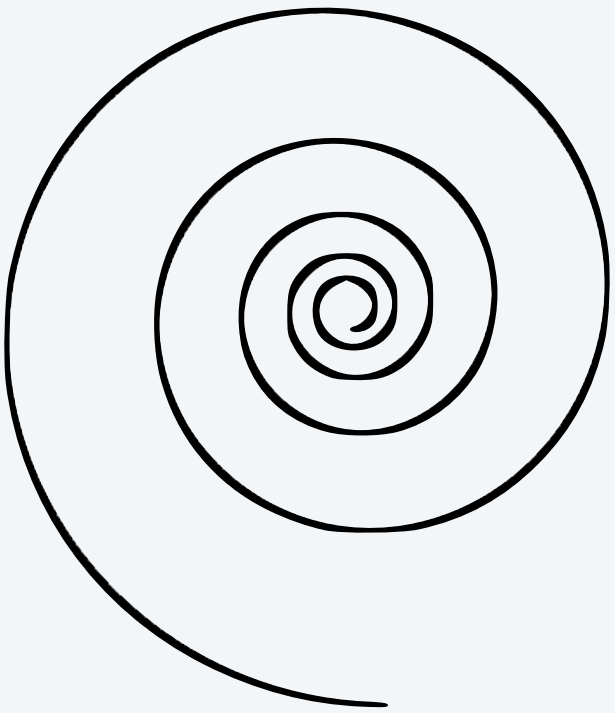
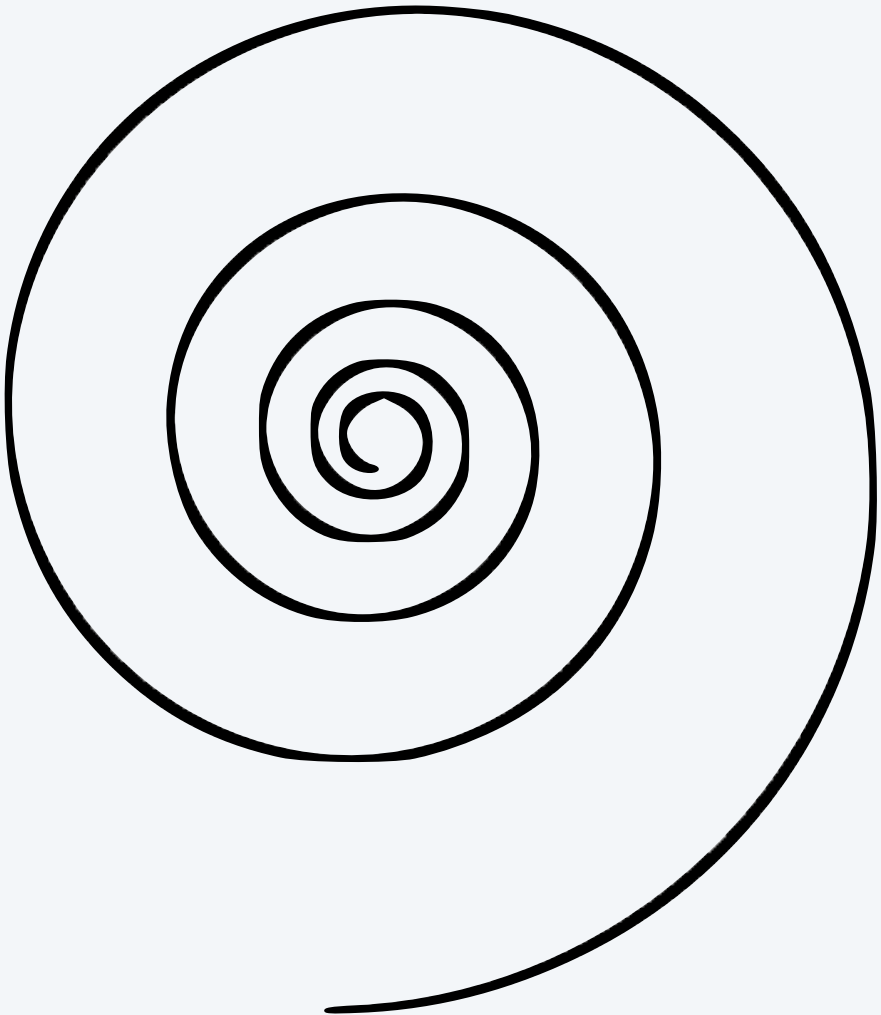
```
% java Spira1 7 1.2
```



```
% java Spira1 1440 1.0004
```



Spira Mirabilis in the wild



Pop quiz 1 on OOP

Q. Fix the serious bug in this code:

```
public class Turtle
{
    private double x, y;
    private double angle;
    public Turtle(double x0, double y0, double a0)
    {
        double x = x0;
        double y = y0;
        double angle = a0;
    }
    ...
}
```

Pop quiz 1 on OOP

Q. Fix the serious bug in this code:

```
public class Turtle
{
    private double x, y;
    private double angle;
    public Turtle(double x0, double y0, double a0)
    {
        double x = x0;
        double y = y0;
        double angle = a0;
    }
    ...
}
```

A. Remove type declarations. They create local variables, which are *different* from the instance variables!

Object-oriented programmers pledge. "I *will not* shadow instance variables"

Every programmer makes this mistake, and it is a difficult one to detect.

COMPUTER SCIENCE

SEDGEWICK / WAYNE

PART I: PROGRAMMING IN JAVA

Image sources

<http://web.media.mit.edu/~papert/>

http://en.wikipedia.org/wiki/Logarithmic_spiral

http://en.wikipedia.org/wiki/Logarithmic_spiral#/media/File:Nautilus_Cutaway_with_Logarithmic_Spiral.png

http://en.wikipedia.org/wiki/File:Low_pressure_system_over_Iceland.jpg

9. Creating Data Types

- Overview
- Point charges
- Turtle graphics
- **Complex numbers**

Crash course in complex numbers

A **complex number** is a number of the form $a + bi$ where a and b are real and $i \equiv \sqrt{-1}$.

Complex numbers are a *quintessential mathematical abstraction* that have been used for centuries to give insight into real-world problems not easily addressed otherwise.

To perform *algebraic operations* on complex numbers, use real algebra, replace i^2 by -1 and collect terms.

- Addition example: $(3 + 4i) + (-2 + 3i) = 1 + 7i$.
- Multiplication example: $(3 + 4i) \times (-2 + 3i) = -18 + i$.

The *magnitude* or *absolute value* of a complex number $a + bi$ is $|a + bi| = \sqrt{a^2 + b^2}$.



Leonhard Euler
1707–1783



A. L. Cauchy
1789–1857

Example: $|3 + 4i| = 5$



Applications: Signal processing, control theory, quantum mechanics, analysis of algorithms...

ADT for complex numbers

A **complex number** is a number of the form $a + bi$ where a and b are real and $i \equiv \sqrt{-1}$.

An **ADT** allows us to write Java programs that manipulate complex numbers.

Values	<i>complex number</i>	$3 + 4i$	$-2 + 2i$
	real part	3.0	-2.0
	imaginary part	4.0	2.0

API (operations)	public class Complex	
	Complex(double real, double imag)	
	Complex plus(Complex b)	<i>sum of this number and b</i>
	Complex times(Complex b)	<i>product of this number and b</i>
	double abs()	<i>magnitude</i>
	String toString()	<i>string representation</i>

Complex number data type implementation: Test client

Best practice. Begin by implementing a simple test client.

```
public static void main(String[] args)
{
    Complex a = new Complex( 3.0, 4.0);
    Complex b = new Complex(-2.0, 3.0);
    StdOut.println("a = " + a);
    StdOut.println("b = " + b);
    StdOut.println("a * b = " + a.times(b));
}
```



```
% java Complex
a = 3.0 + 4.0i
b = -2.0 + 3.0i
a * b = -18.0 + 1.0i
```

↑
What we *expect*, once the implementation is done.

Complex number data type implementation: Instance variables and constructor

Instance variables define data-type values.

Constructors create and initialize new objects.

```
public class Complex
{
    private final double re;
    private final double im;
    public Complex(double real, double imag)
    {
        re = real;
        im = imag;
    }
    ...
}
```

instance variables
are final



Values

<i>complex number</i>	$3 + 4i$	$-2 + 2i$
real part	3.0	-2.0
imaginary part	4.0	2.0

Complex number data type implementation: Methods

Methods define data-type operations (implement APIs).

```

public class Complex
{
    ...
    public Complex plus(Complex b)
    {
        double real = re + b.re;
        double imag = im + b.im;
        return new Complex(real, imag);
    }
    public Complex times(Complex b)
    {
        double real = re * b.re - im * b.im;
        double imag = re * b.im + im * b.re;
        return new Complex(real, imag);
    }
    public double abs()
    { return Math.sqrt(re*re + im*im); }
    public String toString()
    { return re + " + " + im + "i"; }
    ...
}

```

might also write "this.re"
or use Complex a = this

Java keyword "this" is a reference to "this object" and is implicit when an instance variable is directly referenced

$$\begin{aligned}
 a &= v + wi \\
 b &= x + yi \\
 a \times b &= vx + vyi + wxi + wyi^2 \\
 &= vx - wy + (vy + wx)i
 \end{aligned}$$

API

public class Complex	
Complex(double real, double imag)	
Complex plus(Complex b)	<i>sum of this number and b</i>
Complex times(Complex b)	<i>product of this number and b</i>
double abs()	<i>magnitude</i>
String toString()	<i>string representation</i>



Complex number data type implementation

text file named
Complex.java

```
public class Complex
{
    private final double re;
    private final double im;

    public Complex(double real, double imag)
    { re = real; im = imag; }

    public Complex plus(Complex b)
    {
        double real = re + b.re;
        double imag = im + b.im;
        return new Complex(real, imag);
    }
    public Complex times(Complex b)
    {
        double real = re * b.re - im * b.im;
        double imag = re * b.im + im * b.re;
        return new Complex(real, imag);
    }
    public double abs()
    { return Math.sqrt(re*re + im*im); }
    public String toString()
    { return re + " + " + im + "i"; }

    public static void main(String[] args)
    {
        Complex a = new Complex( 3.0, 4.0);
        Complex b = new Complex(-2.0, 3.0);
        StdOut.println("a = " + a);
        StdOut.println("b = " + b);
        StdOut.println("a * b = " + a.times(b));
    }
}
```

instance variables

constructor

methods

test client

```
% java Complex
a = 3.0 + 4.0i
b = -2.0 + 3.0i
a * b = -18.0 + 1.0i
```

The Mandelbrot set

The *Mandelbrot set* is a set of complex numbers.

- Represent each complex number $x + yi$ by a point (x, y) in the plane.
- If a point is *in* the set, we color it BLACK.
- If a point is *not* in the set, we color it WHITE.



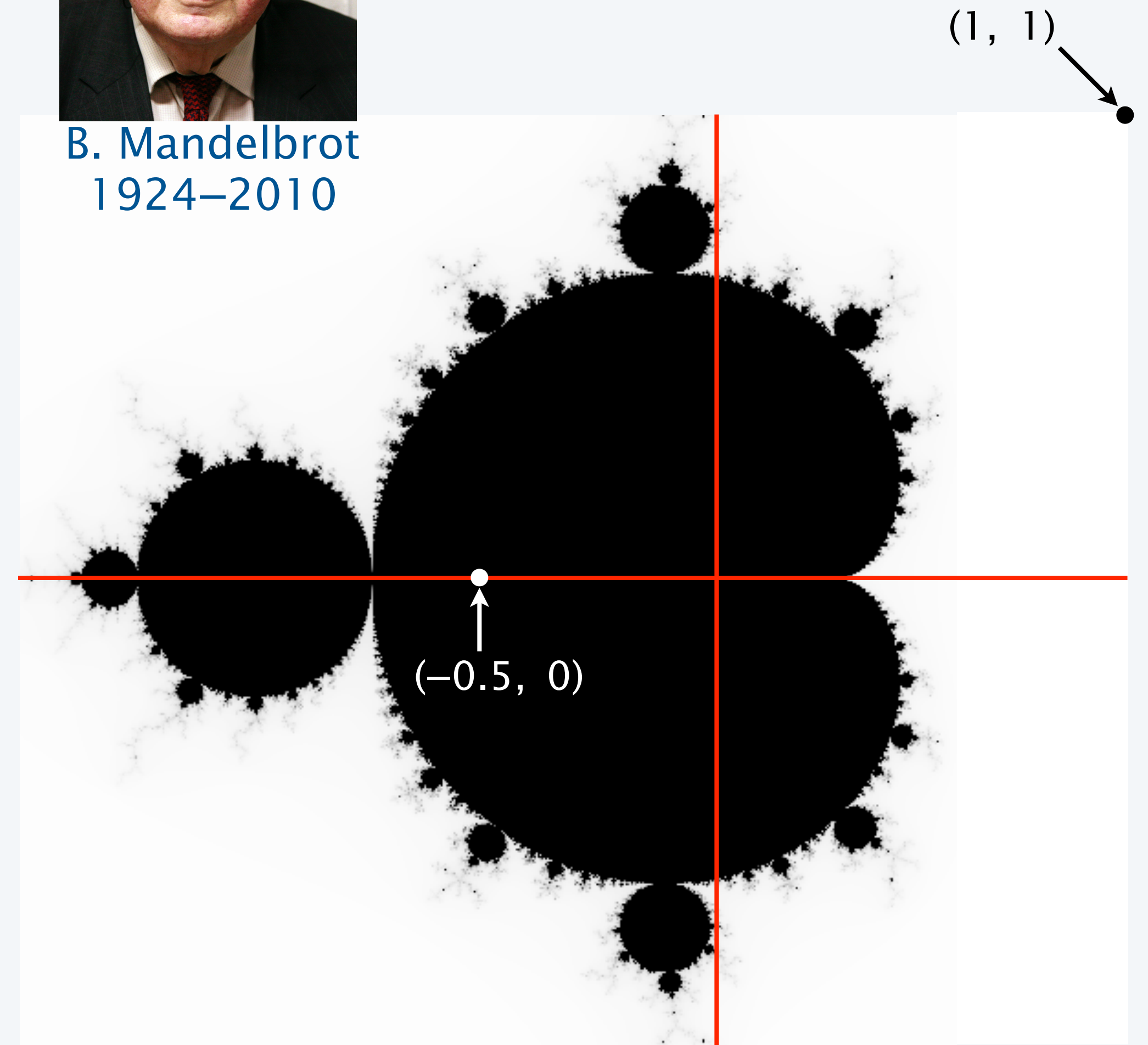
B. Mandelbrot
1924–2010

Examples

- *In* the set: $-0.5 + 0i$.
- *Not* in the set: $1 + i$.

Challenge

- No simple formula exists for testing whether a number is in the set.
- Instead, the set is defined by an *algorithm*.



Determining whether a point is in the Mandelbrot set

Is a complex number z_0 in the set?

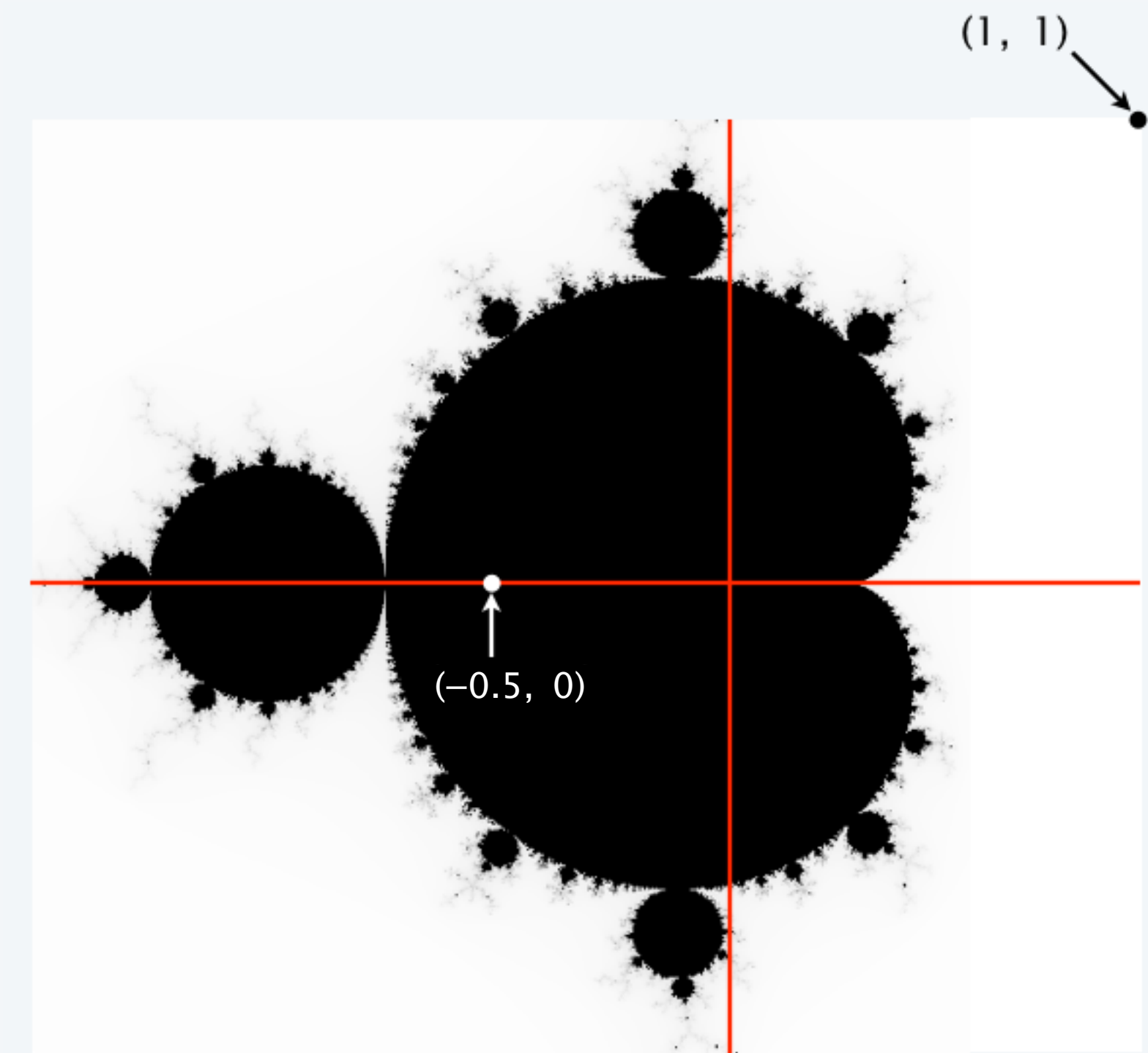
- Iterate $z_{t+1} = (z_t)^2 + z_0$.
- If $|z_t|$ *diverges to infinity*, z_0 is *not in* the set.
- If not, z_0 is *in* the set.

t	z_t
0	$-1/2 + 0i$
1	$-1/4 + 0i$
2	$-7/16 + 0i$
3	$-79/256 + 0i$
4	$-26527/65536 + 0i$

always between $-1/2$ and 0
 $z = -1/2 + 0i$ is *in* the set

t	z_t
0	$1 + i$
1	$1 + 3i$
2	$-7 + 7i$
3	$1 - 97i$
4	$-9407 - 193i$

diverges to infinity
 $z = 1 + i$ is *not in* the set



$$(1+i)^2 + (1+i) = 1 + 2i + i^2 + 1 + i = 1+3i$$

$$(1+3i)^2 + (1+i) = 1 + 6i + 9i^2 + 1 + i = -7+7i$$

Plotting the Mandelbrot set

Practical issues

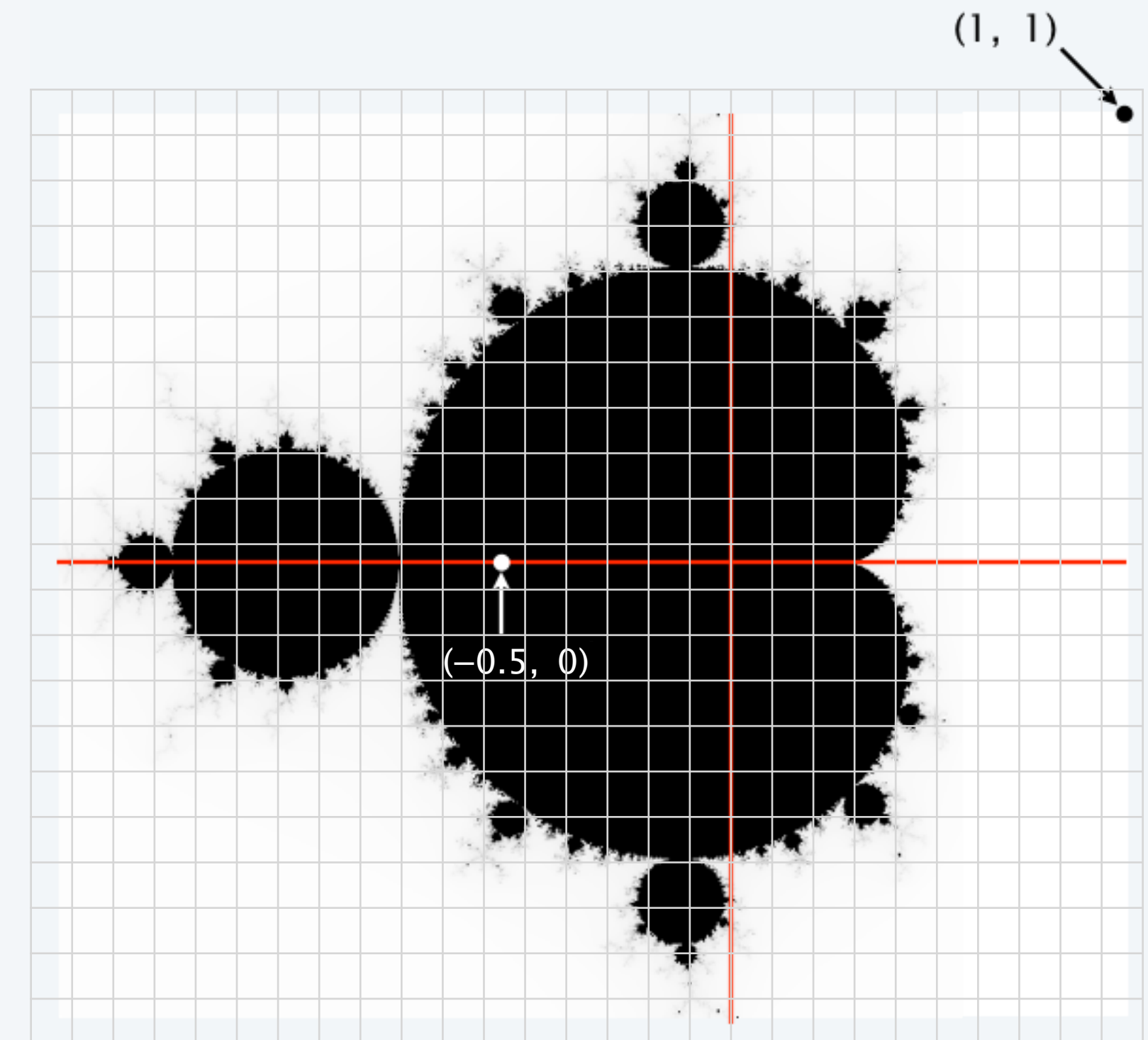
- Cannot plot infinitely many points.
- Cannot iterate infinitely many times.

Approximate solution for first issue

- Sample from an N -by- N grid of points in the plane.
- Zoom in to see more detail (stay tuned!).

Approximate solution for second issue

- Fact: if $|z_t| > 2$ for any t , then z is *not* in the set.
- Pseudo-fact: if $|z_{255}| \leq 2$ then z is "likely" in the set.



Important note: Solutions imply significant computation.

Complex number client: Mandelbrot set visualization (helper method)

Mandelbrot function of a complex number.

- Returns WHITE if the number is not in the set.
- Returns BLACK if the number is (probably) in the set.

```
public static Color mand(Complex z0)
{
    Complex z = z0;
    for (int t = 0; t < 255; t++)
    {
        if (z.abs() > 2.0) return Color.WHITE;
        z = z.times(z);
        z = z.plus(z0);
    }
    return Color.BLACK;
}
```

For a more dramatic picture,
return new Color(255-t, 255-t, 255-t)
or colors picked from a color table.

Complex number client: Mandelbrot set visualization

```
import java.awt.Color;
public class Mandelbrot
{
    public static Color mand(Complex z0)
    { // See previous slide. }
    public static void main(String[] args)
    {
        double xc    = Double.parseDouble(args[0]);
        double yc    = Double.parseDouble(args[1]);
        double size  = Double.parseDouble(args[2]);
        int N       = Integer.parseInt(args[3]);
        Picture pic  = new Picture(N, N);

        for (int col = 0; col < N; col++)
            for (int row = 0; row < N; row++)
            {
                double x0 = xc - size/2 + size*col/N;
                double y0 = yc - size/2 + size*row/N;
                Complex z0 = new Complex(x0, y0);
                Color color = mand(z0);
                pic.set(col, N-1-row, color);
            }
        pic.show();
    }
}
```

scale to screen coordinates

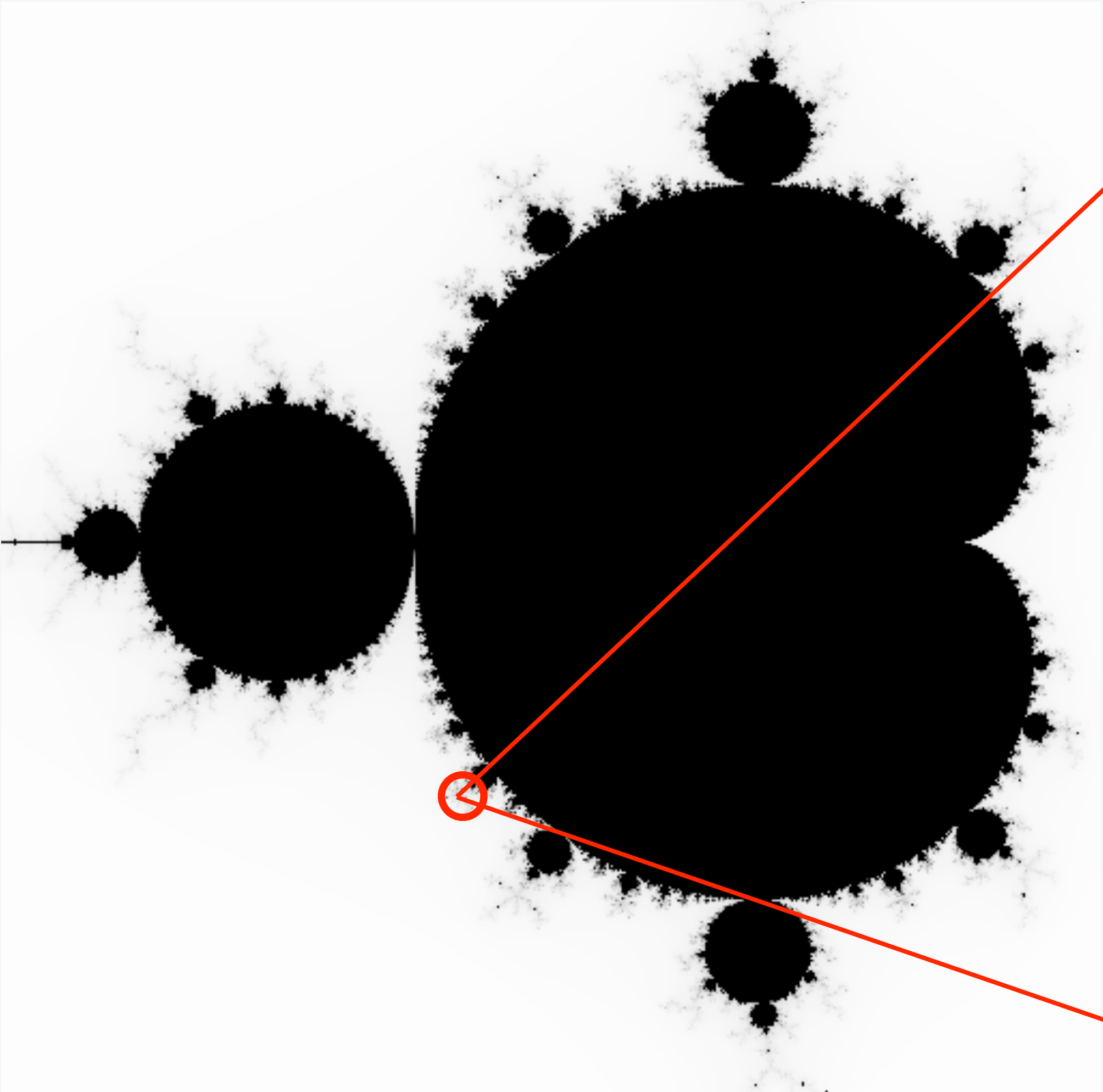
(0, 0) is upper left corner

```
% java Mandelbrot -.5 0 2 32
```

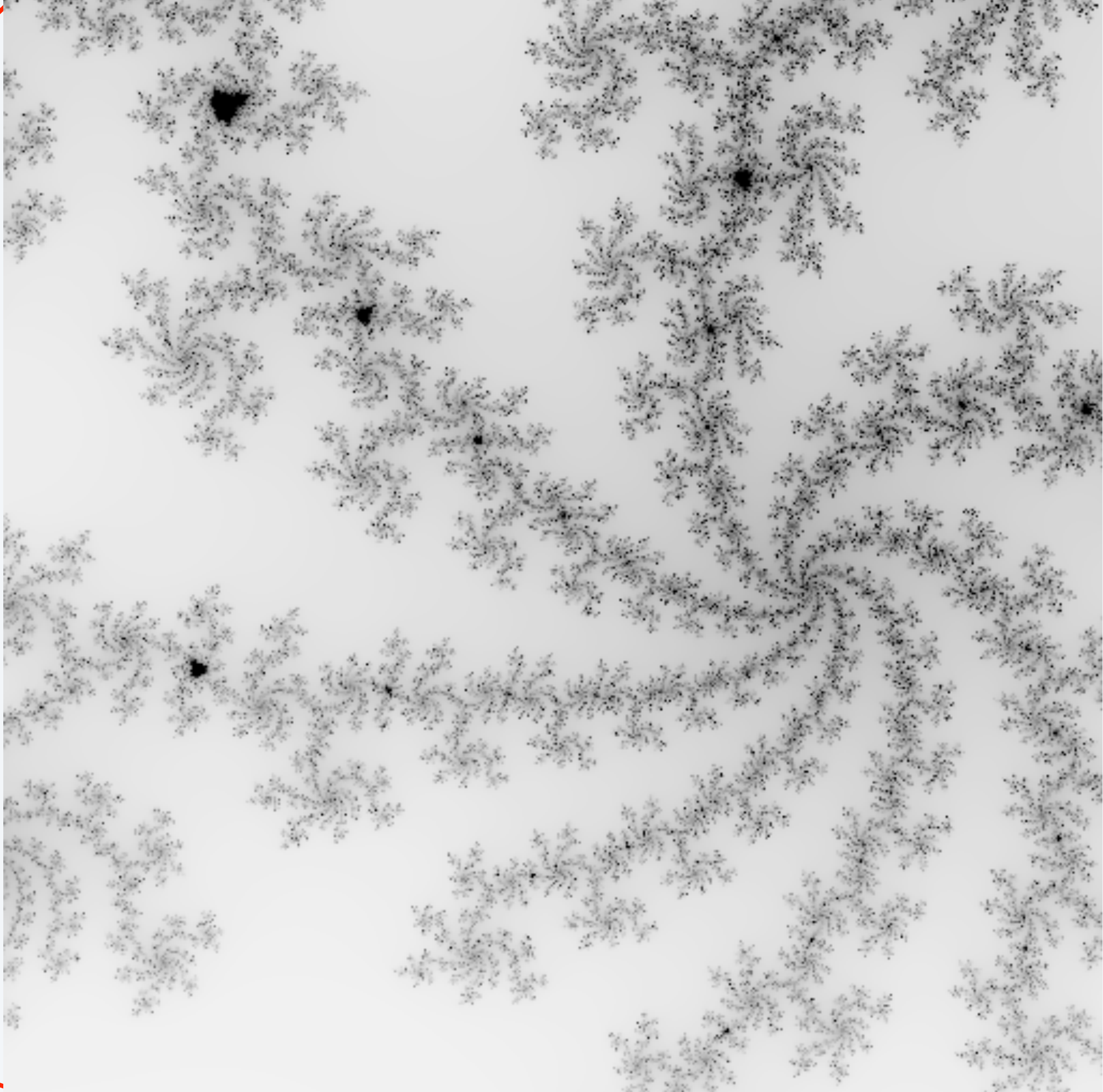


Mandelbrot Set

```
% java GrayscaleMandelbrot -.5 0 2
```



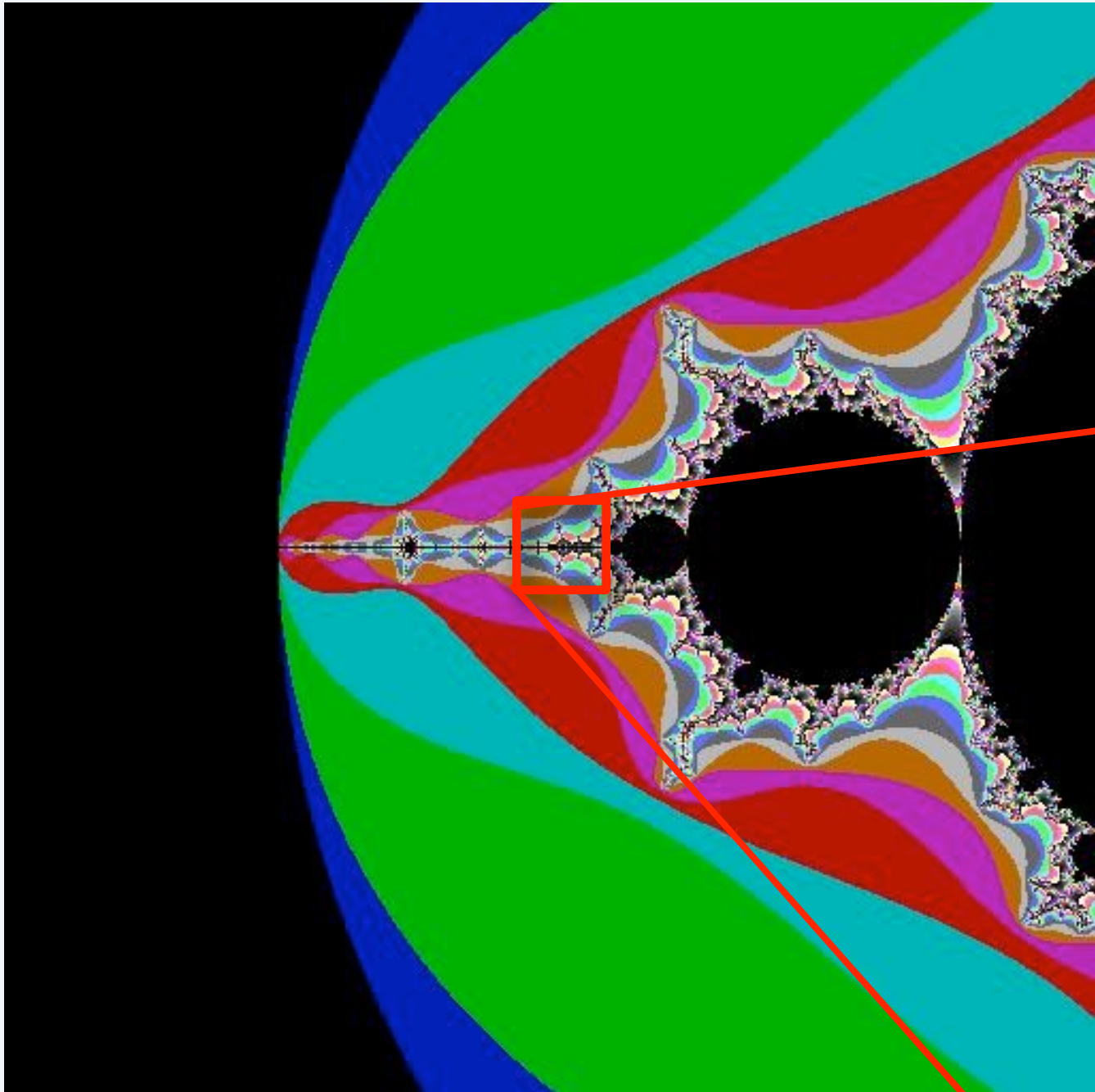
```
% java GrayscaleMandelbrot .1045 -.637 .01
```



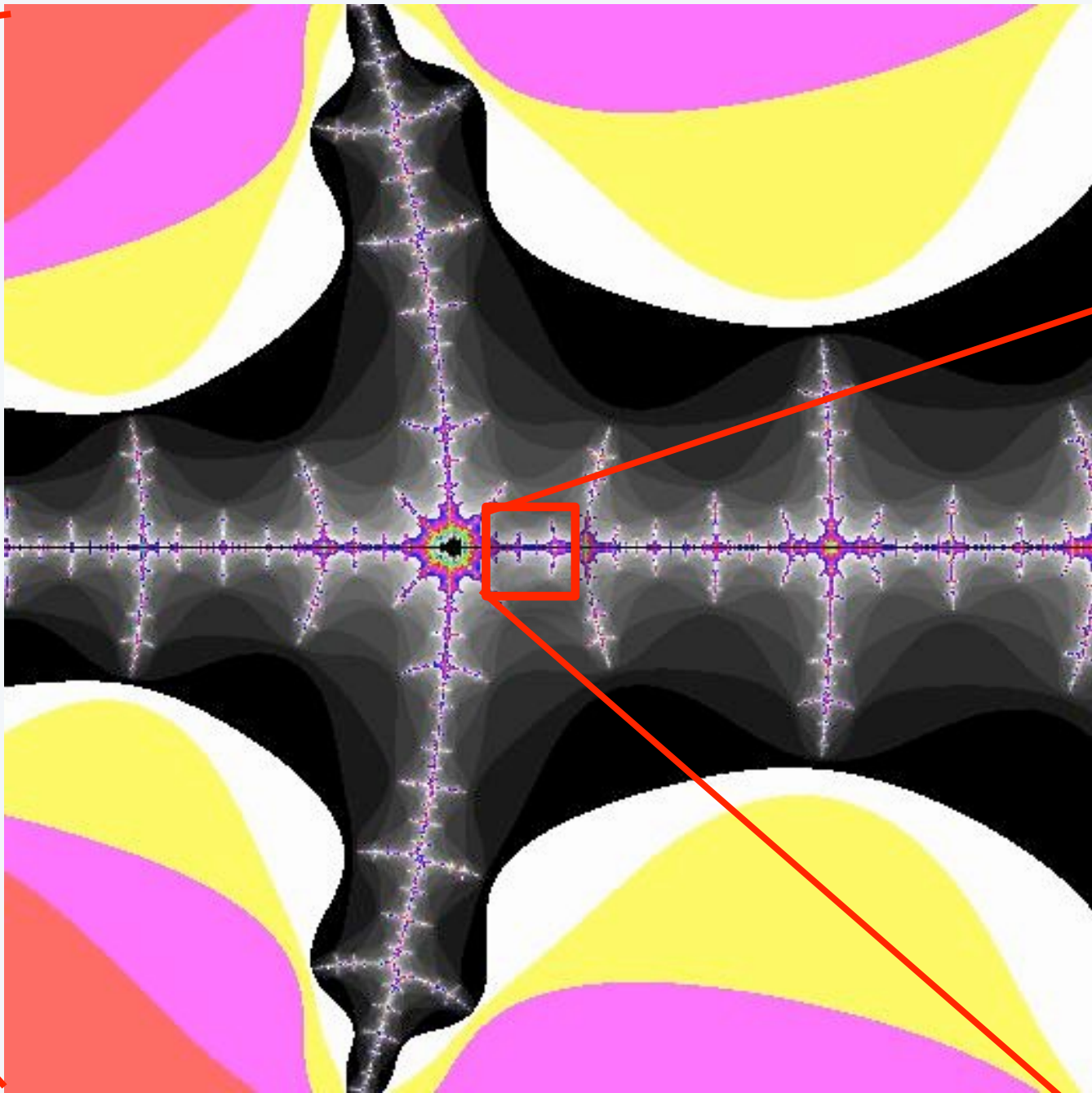
Mandelbrot Set

```
% java ColorMandelbrot -1.5 0 2 < mandel.txt
```

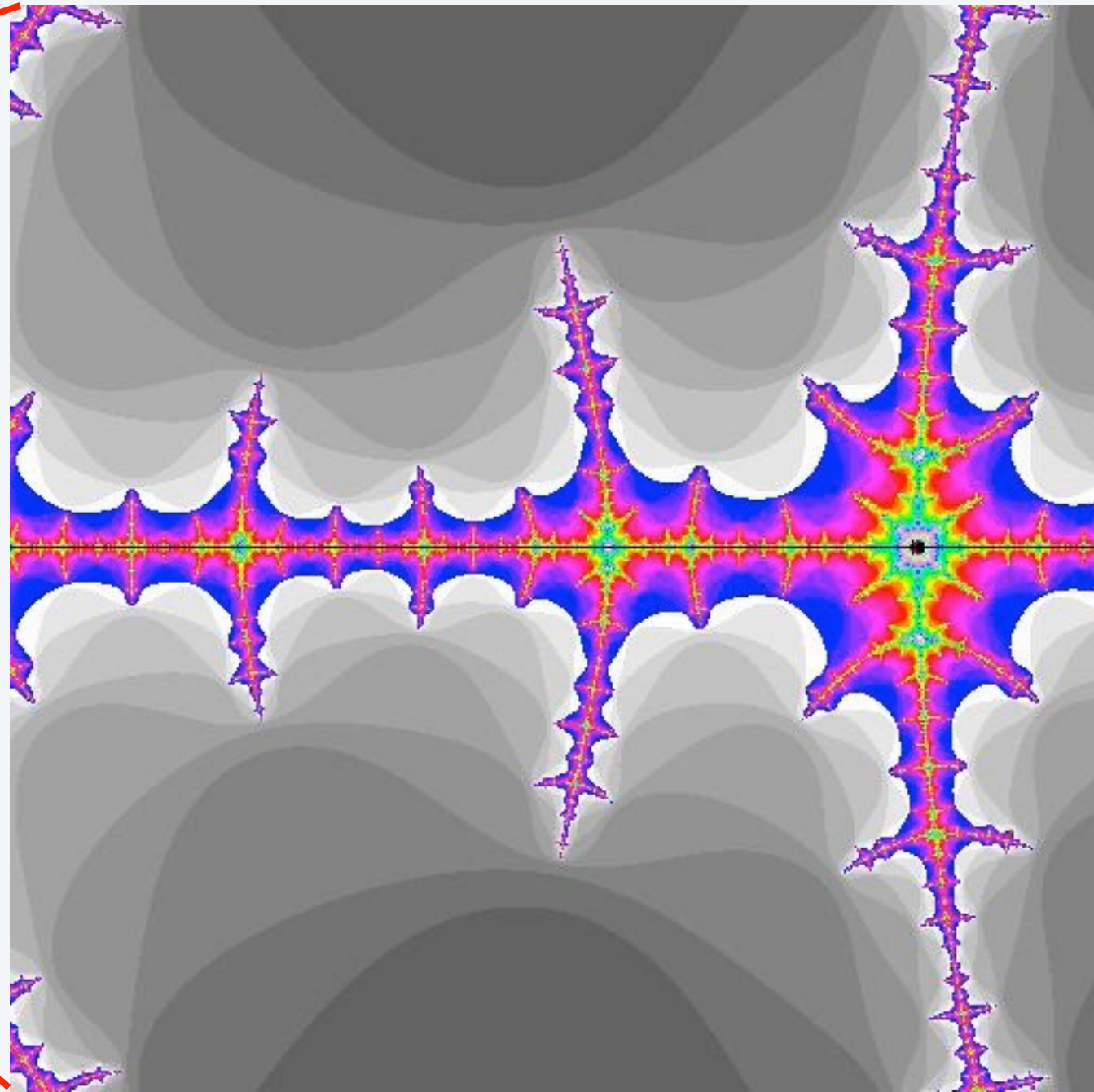
color map

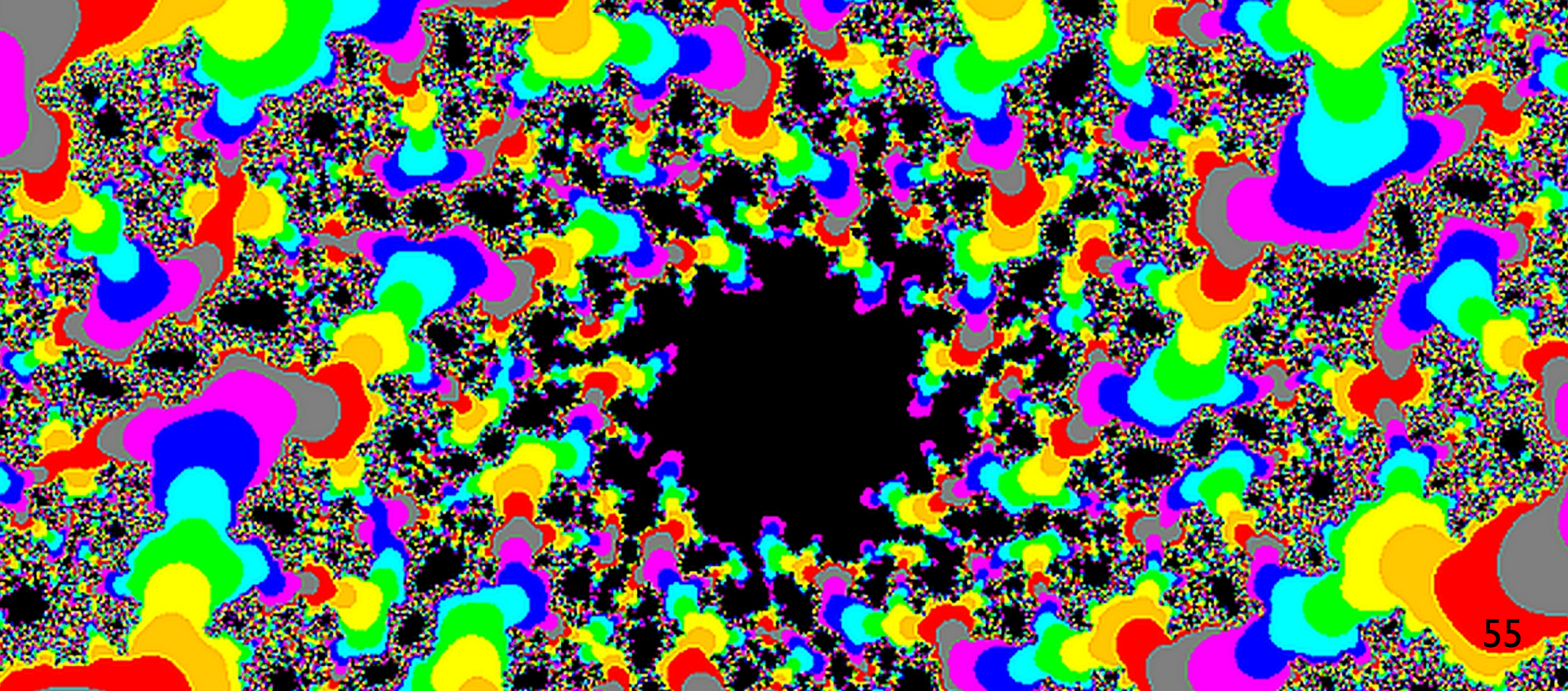
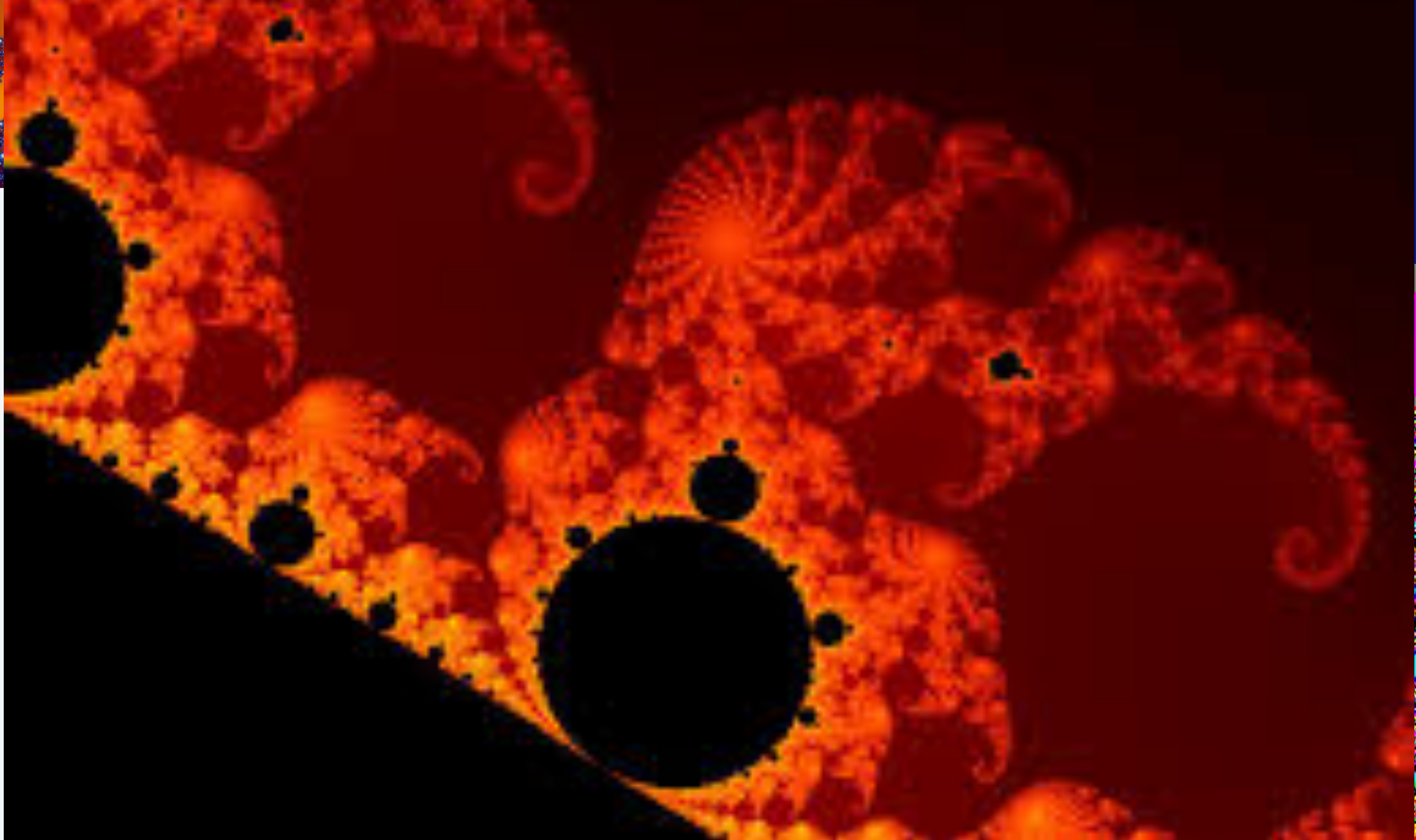
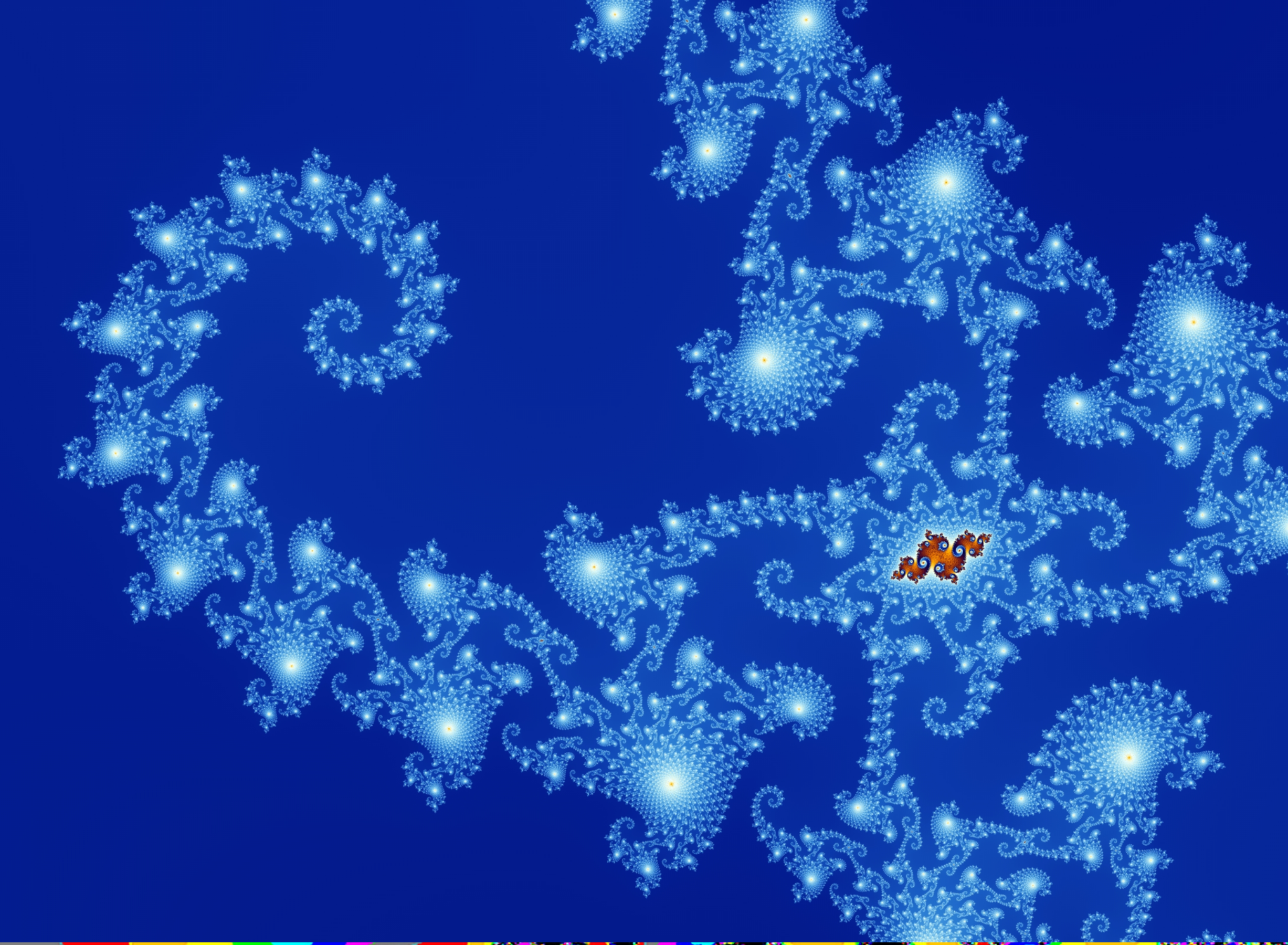
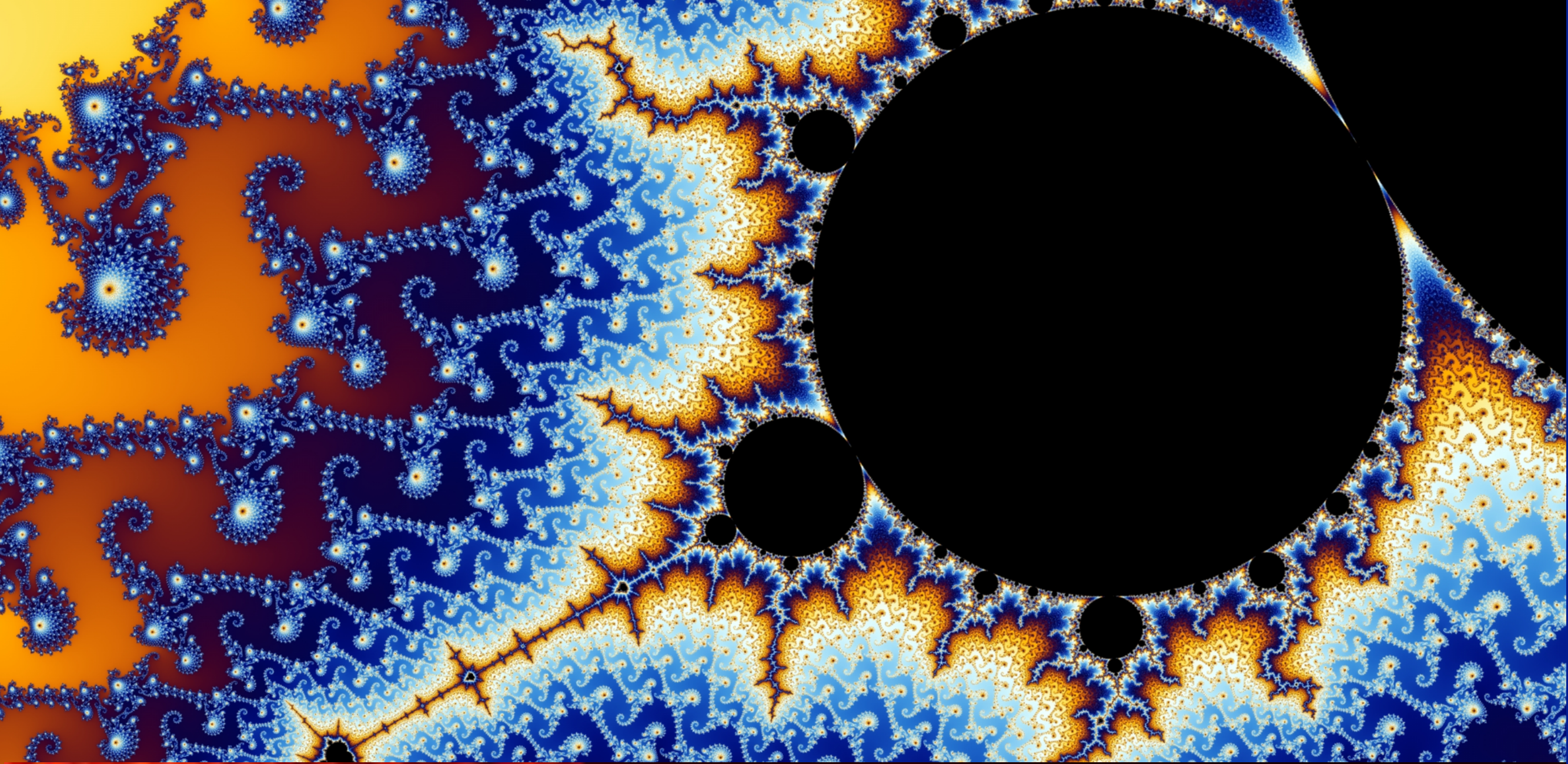


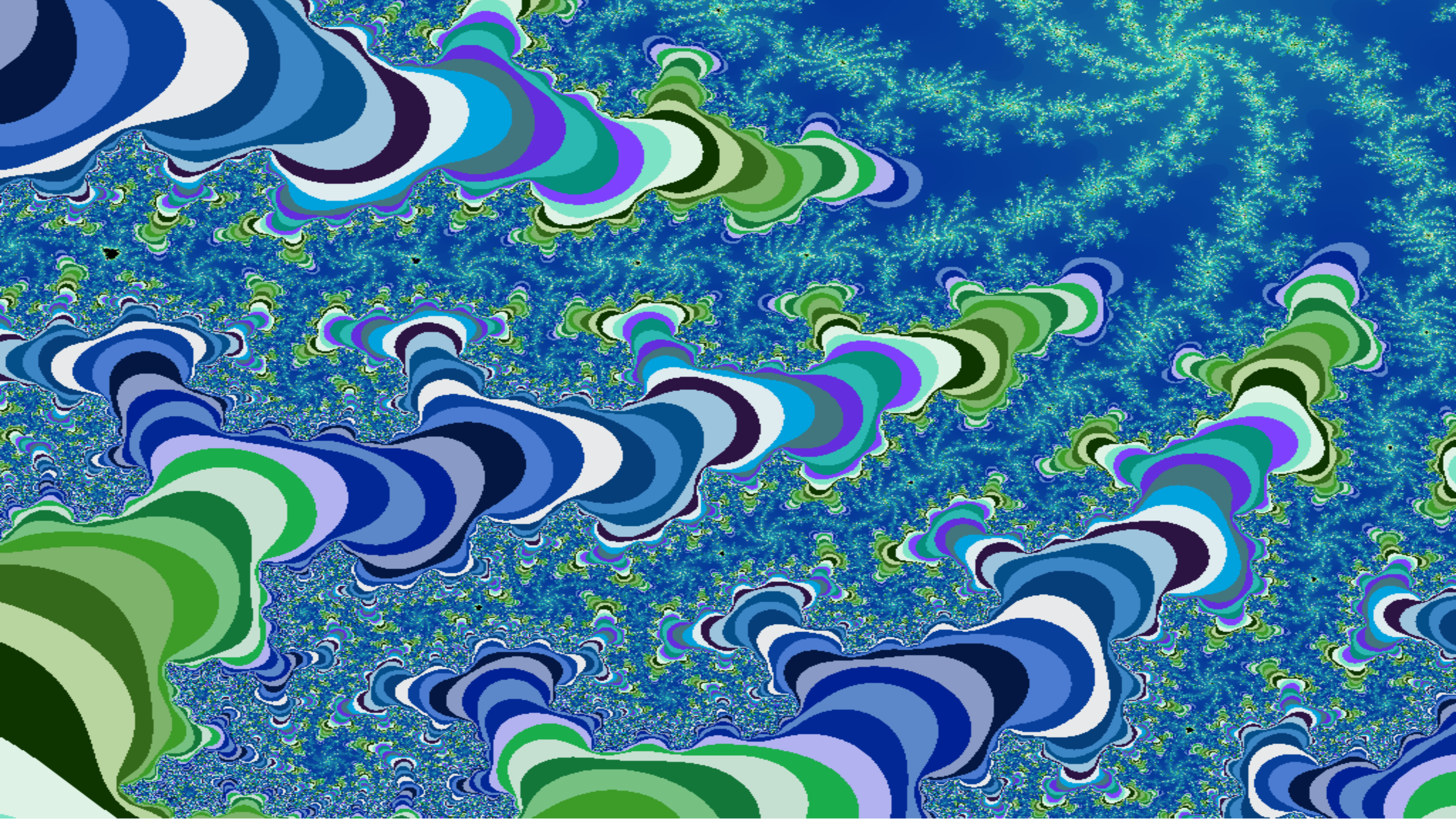
```
-1.5 0 2
```



```
-1.5 0 .002
```



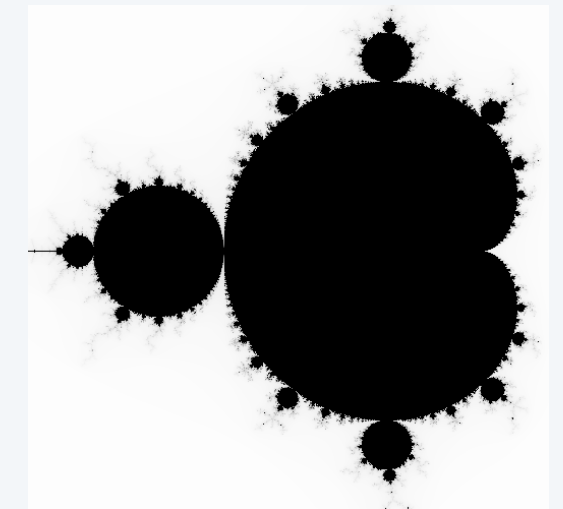




OOP summary

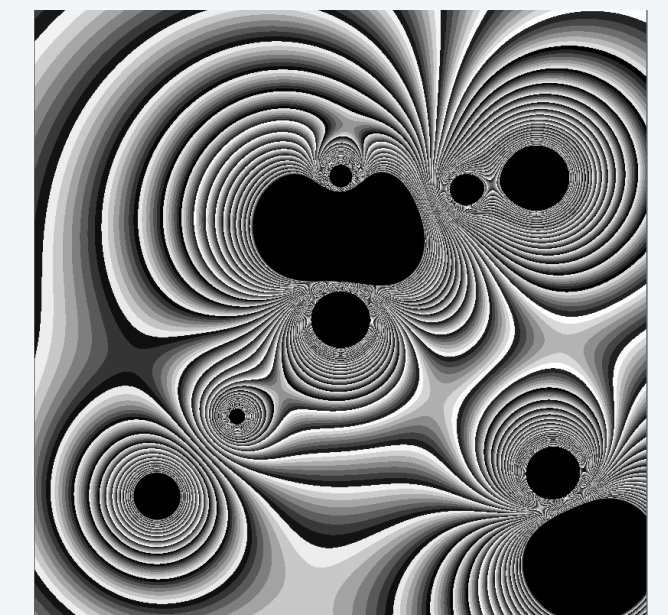
Object-oriented programming (OOP)

- Create your own data types (sets of values and ops on them).
- Use them in your programs (manipulate *objects*).



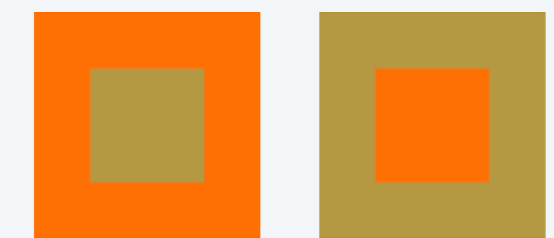
OOP helps us simulate the physical world

- Java objects model real-world objects.
- Not always easy to make model reflect reality.
- Examples: charged particle, color, sound, genome....



OOP helps us extend the Java language

- Java doesn't have a data type for every possible application.
- Data types enable us to add our own abstractions.
- Examples: complex, vector, polynomial, matrix, picture....

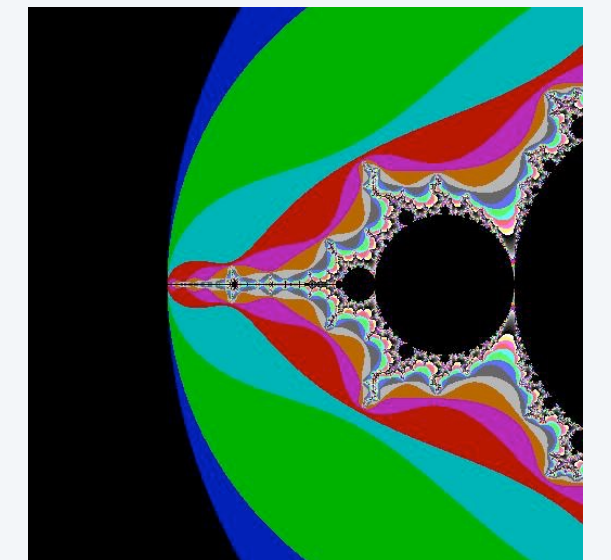
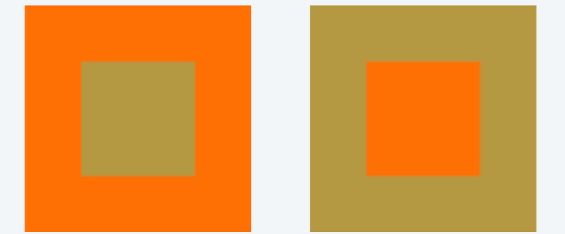
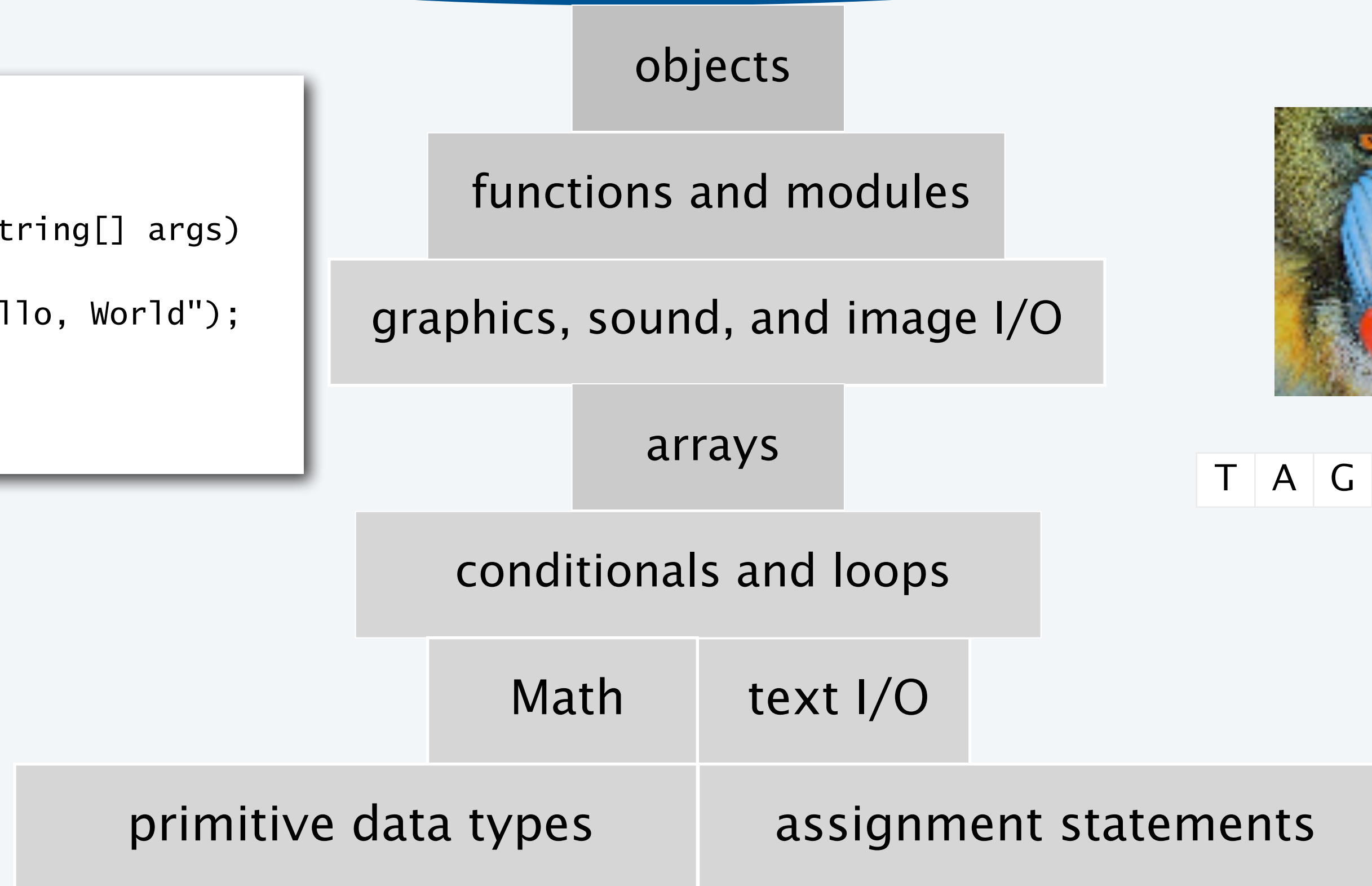


T A G A T G T G C T A G C

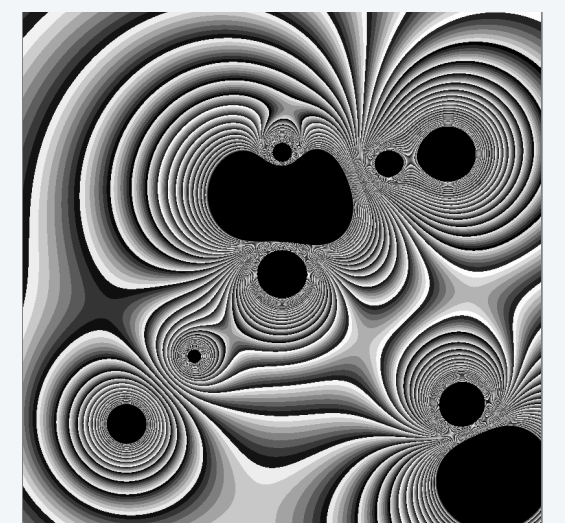
You have come a long way

any program you might want to write

```
public class HelloWorld
{
    public static void main(String[] args)
    {
        System.out.println("Hello, World");
    }
}
```



T A G A T G T G C T A G C



Course goal. Open a *whole new world* of opportunity for you (programming).



Image sources

http://en.wikipedia.org/wiki/Leonhard_Euler#/media/File:Leonhard_Euler.jpg

http://en.wikipedia.org/wiki/Augustin-Louis_Cauchy

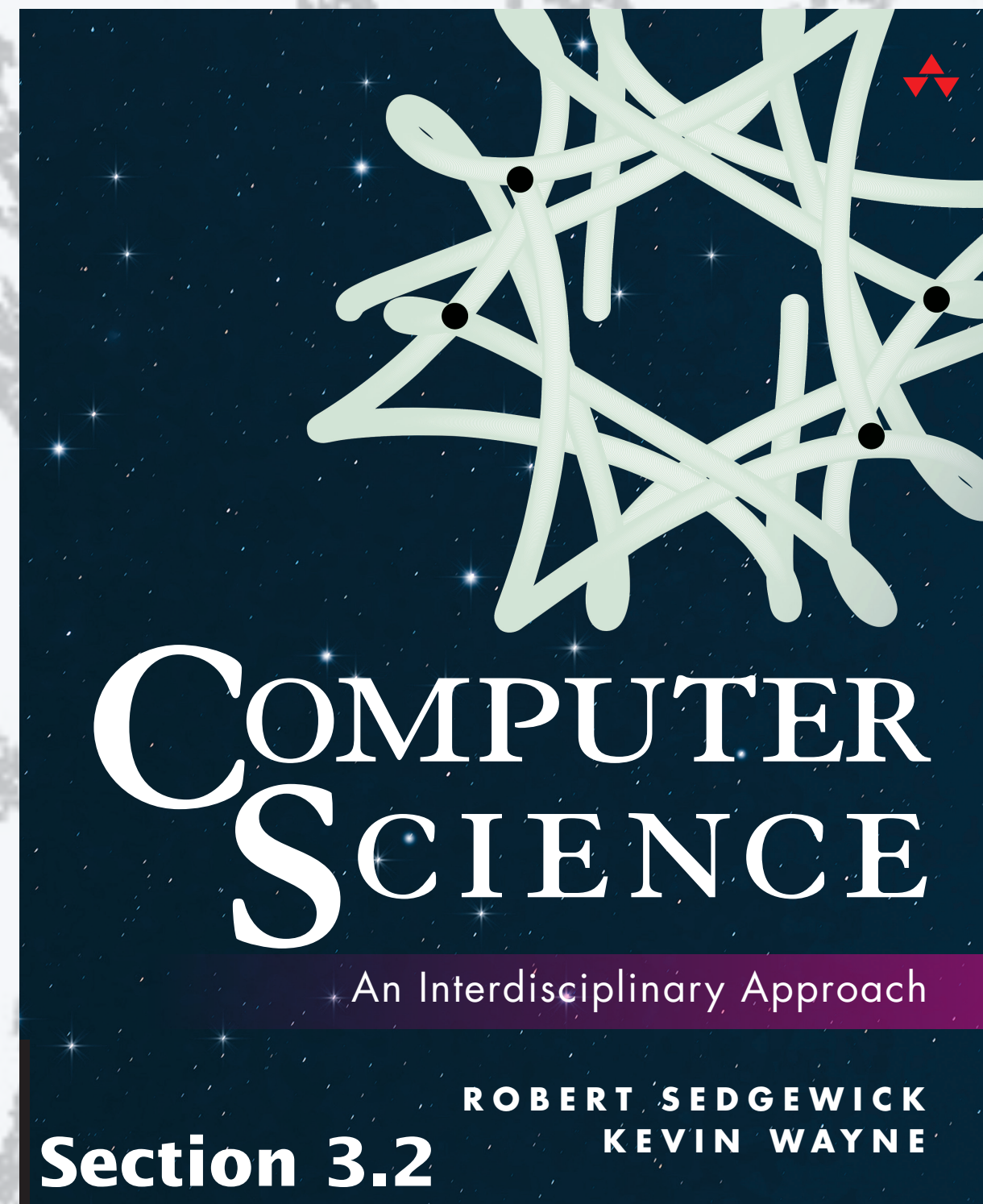
http://upload.wikimedia.org/wikipedia/commons/e/e9/Benoit_Mandelbrot_mg_1804-d.jpg

http://upload.wikimedia.org/wikipedia/commons/f/fc/Mandel_zoom_08_satellite_antenna.jpg

<http://upload.wikimedia.org/wikipedia/commons/1/18/Mandelpart2.jpg>

http://upload.wikimedia.org/wikipedia/commons/f/fb/Mandel_zoom_13_satellite_seehorse_tail_with_julia_island.jpg

http://upload.wikimedia.org/wikipedia/commons/4/44/Mandelbrot_set_à_la_Pop_Art_-_Wacker_Art_Fractal_Generator.jpg



9. Creating Data Types

<http://introcs.cs.princeton.edu>