Linear algebra is one of the topics covered by the GRE test in mathematics. Here are the questions relating to linear algebra on the sample test.

3. If $V$ and $W$ are 2-dimensional subspaces of $\mathbb{R}^4$, what are the possible dimensions of the subspace $V \cap W$?

(A) 1 only  (B) 2 only  (C) 0 and 1 only  (D) 0, 1, and 2 only  (E) 0, 1, 2, 3, and 4

12. Let $A$ be a $2 \times 2$ matrix for which there is a constant $k$ such that the sum of the entries in each row and each column is $k$. Which of the following must be an eigenvector of $A$?

I. \[
\begin{bmatrix}
1 \\
0
\end{bmatrix}
\]

II. \[
\begin{bmatrix}
0 \\
1
\end{bmatrix}
\]

III. \[
\begin{bmatrix}
1 \\
1
\end{bmatrix}
\]

(A) I only  (B) II only  (C) III only  (D) I and II only  (E) I, II, and III

18. Let $V$ be the real vector space of all real $2 \times 3$ matrices, and let $W$ be the real vector space of all real $4 \times 1$ column vectors. If $T$ is a linear transformation from $V$ onto $W$, what is the dimension of the subspace \{ $v \in V : T(v) = 0$ \}?

(A) 2  (B) 3  (C) 4  (D) 5  (E) 6

27. Consider the two planes $x + 3y - 2z = 7$ and $2x + y - 3z = 0$ in $\mathbb{R}^3$. Which of the following sets is the intersection of these planes?

(A) $\emptyset$  
(B) \{(0, 3, 1)\}  
(C) \{(x, y, z) : x = t, y = 3t, z = 7 - 2t, t \in \mathbb{R}\}  
(D) \{(x, y, z) : x = 7t, y = 3 + t, z = 1 + 5t, t \in \mathbb{R}\}  
(E) \{(x, y, z) : x - 2y - z = -7\}
36. Let $M$ be a $5 \times 5$ real matrix. Exactly four of the following five conditions on $M$ are equivalent to each other. Which of the five conditions is equivalent to NONE of the other four?

(A) For any two distinct column vectors $u$ and $v$ of $M$, the set $\{u, v\}$ is linearly independent.
(B) The homogeneous system $Mx = 0$ has only the trivial solution.
(C) The system of equations $Mx = b$ has a unique solution for each real $5 \times 1$ column vector $b$.
(D) The determinant of $M$ is nonzero.
(E) There exists a $5 \times 5$ real matrix $N$ such that $NM$ is the $5 \times 5$ identity matrix.

50. Let $A$ be a real $2 \times 2$ matrix. Which of the following statements must be true?

I. All of the entries of $A^2$ are nonnegative.
II. The determinant of $A^2$ is nonnegative.
III. If $A$ has two distinct eigenvalues, then $A^2$ has two distinct eigenvalues.

(A) I only  (B) II only  (C) III only  (D) II and III only  (E) I, II, and III

58. Suppose $A$ and $B$ are $n \times n$ invertible matrices, where $n > 1$ and $I$ is the $n \times n$ identity matrix. If $A$ and $B$ are similar matrices, which of the following statements must be true?

I. $A - 2I$ and $B - 2I$ are similar matrices.
II. $A$ and $B$ have the same trace.
III. $A^{-1}$ and $B^{-1}$ are similar matrices.

(A) I only  (B) II only  (C) III only  (D) I and III only  (E) I, II, and III

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