Math 105 History of Mathematics
Second test
May 2015

You may refer to two sheets of notes on this test. Points for each problem are in square brackets. You may do the problems in any order that you like, but please start your answers to each problem on a separate page of the bluebook. Please write or print clearly.

1. [27] Essay. Please think about these topics and make an outline before you begin writing. You will be graded on how well you present your ideas as well as your ideas themselves. Each essay should be relatively short—one to three written pages. There should be no fluff in your essays. Make your essays well-structured and your points as clearly as you can.

Topic A. One quality of the mathematics of ancient India and China, on the one hand, which differs from that of Greek and Islamic mathematicians, on the other hand, is formalism. Here formalism means careful definitions and clear proofs. (Another aspect of formalism which doesn't have much role here is symbolism, as in symbolic algebra which wasn't developed until later.) Explain what this difference is. Then describe the effect of this difference had on the development of mathematics in those cultures.

Topic B. The arithmetic triangle, commonly known as Pascal's triangle. Which of the several regions that we've studied—Greece, India, China, Islamic/Arabic, and western Europe—was the arithmetic triangle known? How was it used in those regions which knew it? To what degree was the knowledge of the aritmetic triangle passed among these regions? Back up your conclusions with evidence. Clarify the degree that you are certain of your conclusions, since, in some cases, the evidence is clear, in others, it may be circumstantial.

Topic C. Albert Girard first stated (but did not prove) the fundamental theorem of algebra about roots of polynomials in the early 17th century. His form of the theorem also included a relation between the coefficients of the polynomial and the roots as well as the number of roots. First state the fundamental theorem of algebra. Then explain the relation between the coefficients and roots of the polynomial. To illustrate it, you could show how the coefficientes of the cubic polynomial $x^3 - 7x^2 + 14x - 8$ are related to its three roots 1, 2, and 4.

2. [20] A "Chinese remainder problem" was first described in Sun Zi's *Mathematical Manual* in the third century, and complete solutions to problems of this type were given by Qin Jiushao in the 13th century. Solve the following problem of this type: find an integer N whose remainder on division by 15 is 4 and whose remainder on division by 7 is 5. In modern notation, this would read

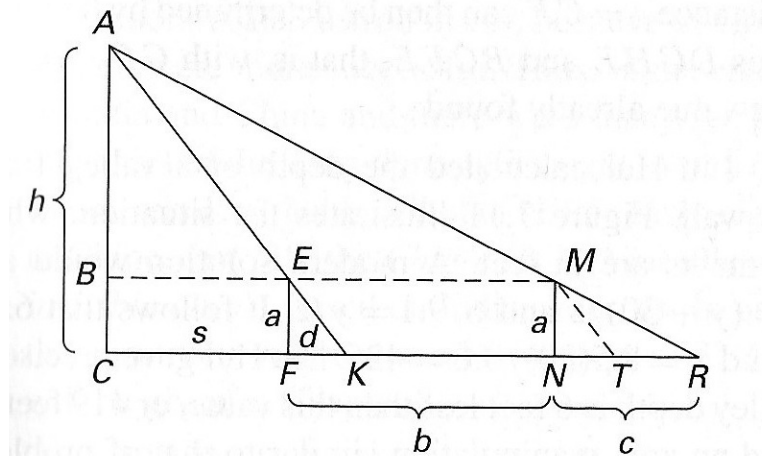
$$\begin{aligned} N &\equiv 4 \pmod{15} \\ N &\equiv 5 \pmod{7} \end{aligned}$$

Show your computations and/or explain how you derived the answer.

3. [20] On remote measurement. We've seen solutions to problems of remote measurement in several different cultures. One particular problem that occurred in both China and India was to determine the distance and height of a remote mountain, and the solution was similar.

Here's a problem of that type. It's the first problem in Lui Hui's *Sea Island Matheamtical Manual*, third century CE.

Erect two $a = 5$ foot tall poles on flat land in a line with the mountain, the back one $b = 1000$ feet behind the front one. For the front pole, a point on the ground $d = 123$ feet behind the front pole aligns with the top of the pole and the mountain top, but for the back pole, a point $c = 127$ feet behind it aligns with the top of that pole and the mountain top. Refer to the diagram.



Determine the height y (h in the diagram) of the mountain and the distance x (s in the diagram) from the front pole to (the point below the top of) the mountain. That is, find expressions for x and y in terms of the known quantities a, b, c, and d. If you like, you can find x and y numerically for the values $a = 5$, $b = 1000$, $c = 127$, and $d = 123$, but that's not necessary.

Describe what principles of geometry you used. Were those principles known in China and/or India?

4. [15] Short essay. On logarithms. One of the interesting subjects during the renaissance was the "double array" which included one geometric array of numbers and one arithmetic array of numbers. For example:

1	3	9	27	81	243	729	2187	6561
0	1	2	3	4	5	6	7	8

Explain how this double array can be used to simplify the process of multiplication. Illustrate your explanation by multiplying 27 by 243 using the double array.

Burgi produced a table of logarithms using just such a double array. He took as the ratio in the geometric array the number 1.0001. Explain why he used a number so close to 1.

5. [18; 3 points each part] True/false. For each sentence write the whole word "true" or the whole word "false". If it's not clear whether it should be considered true or false, you may explain in a sentence if you prefer.

a. The parallel postulate was proven to be independent of the rest of Euclidean geometry by the geometer al-Khwarizmi.

b. Aryabhata, an Indian mathematician, expressed his mathematics in verse.

c. The Mayans had a base 20 place value system for their numbers.

d. Oresme's developed a version of fundametal theorem of calculus that said that if you erect a perpendicular line at each point on a line representing time so that the length of that line is proportional to the velocity at each instant of time, then the area of the resulting region is proportional to the distance travelled.

e. Negative numbers were accepted in China and India long before they were accepted by the Europeans.

f. Projective geometry is the study of the motion of projectiles such a ball fired from a cannon.