Math 114 Discrete Math

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Your name: ____________________________

You may use two sheets of prepared notes during this test. Show all your work for credit. You may leave your answers as expressions such as $\binom{8}{3} .31^3\, .69^5$ if you like. Points for each problem are in square brackets.

Problem 1. On mathematical induction. [16] Let a function f be defined recursively on positive integers by $f(1) = 0$, $f(2) = 3$, and $f(n) = 2f(n-1) - f(n-2) + 4$ for $n \geq 3$. Using this definition, prove by induction that $f(n) = 2n^2 - 3n + 1$. In your proof, identify the base case(s) and the inductive step.

Problem 2. On Euclid's algorithm. [10] Reduce the common fraction $\frac{403}{481}$ to lowest terms. (This involves finding the greatest common divisor of the numerator and denominator.) Show your work.

Problem 3. On binomial coefficients. [10] Find the coefficient of x^4y^7 in $(x+y)^{11}$. (Show your work.)

Problem 4. On the pigeonhole principle. [12] Prove that at a party where there are at least two people, there are at least two people who know the same number of people. Assume that if person A knows person B, then person B also knows person A. Furthermore, assume that each person A knows him/herself. (Suggestions: Let n be the number of people at the party. Consider the number of people that a given person knows. What is the range of values of that number? What happens if some one person knows everyone? What happens if no one knows everyone?)

Problem 5. Principles of combinatorics [20; 5 points each part] How many strings of five decimal digits

a. begin with an odd digit?

b. do not contain the same digit twice?

c. have exactly three digits that are 9s?

d. contain exactly three distinct digits?

Problem 6. Finite probability. [10; 5 points each part] A fair coin is flipped eight times.

a. What is the probability that there are exactly 3 heads.

b. What is the probability that there are at least 3 heads?

Problem 7. Probability theory. [12; 6 points each part] Suppose that E and F are events such that $p(E) = 0.8$ and $p(F) = 0.6$. (Do not assume that these are independent events.)

a. Show that the probability of their union, $P(E \cup F)$, is at least 0.8.

b. Show that the probability of their intersection, $P(E \cap F)$, is at least 0.4.

Problem 8. On expected value. [10; 5 points each part] A random variable X takes on the values 1 through 5 with the indicated values.

1	2	3	4	5
.4	.2	.1	.2	.1

a. Compute its expectation $E(X)$.

b. Compute the expectation of its square, $E(X^2)$.

#1.[16]	
#2.[10]	
#3.[10]	
#4.[12]	
#5.[20]	
#6.[10]	
#7.[12]	
#8.[10]	
Total	