Math 114 Discrete Math Midterm

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Your name: ______________________________

You may use one sheet of prepared notes during this test. Show all your work for credit. Points for each problem are in square brackets.

Problem 1. Translation into symbolic expressions. [20; 5 points each part] Let P, Q, and R be abbreviations for the following predicates.

$P(x)$: x is perfect
$Q(x)$: x failed the quiz
$R(x)$: x read a lot of books

Write these propositions using P, Q, R, logical connectives, and quantifiers.

a. Everyone who is perfect read a lot of books.

b. No one who read a lot of books failed the quiz.

c. Someone who failed the quiz isn't perfect.

d. If everyone reads a lot of books, then no one will fail the quiz.

Problem 2. On truth tables. [20; 10 point each part]

a. Use a truth table to show that $(p \to q) \wedge (p \to r)$ is logically equivalent to $p \to (q \wedge r)$. Explain in a sentence why your truth table shows that they are logically equivalent.

b. Use a truth table to show that $(p \to q) \vee (p \to r)$ is not logically equivalent to $p \to (q \vee r)$. Explain in a sentence why your truth table shows that they aren't logically equivalent.

Problem 3. Interpretation of symbolic expressions. [20; 5 points each part] Determine the truth value of each of the following statements if the universe of discourse for each variable consists of all real numbers. Simply write "true" or "false" for each; no need to explain why.

__________ **a.** $\forall x\, \exists y\, (y = 3x + 2)$.

__________ **b.** $\exists y\, \forall x\, (y = 3x + 2)$.

__________ **c.** $\exists x\, \exists y\, (x^2 + y^2 = 1)$.

__________ **d.** $\forall y\, \forall x\, (x + y)^2 = x^2 + y^2$.

__________ **e.** $\forall x\, (x < 0 \lor x = 0 \lor x > 0)$.

Problem 4. On rules of inference. [20] Given statements 1 and 2, find an argument to conclude statement n. For each intermediate statement you make, state what rule of inference you use and the number(s) of the previous lines that rule uses.

(You don't have to use symbolic notation for this problem, but it may help. Also, if you don't remember the name for a rule of inference, then just state the whole rule symbolically.)

1. Linda, a student in this class, owns a red convertible.

2. Everyone who owns a red convertible has gotten at least one speeding ticket.

n. Someone in this class has gotten a speeding ticket.

Problem 5. On proofs. [20] Recall that an integer n is *even* iff $\exists k, n = 2k$. Also, an integer n is *odd* iff $\exists k, n = 2k + 1$. Furthermore, each integer is either even or odd, but no integer is both even and odd.

a. [15] Show that if n^2 is an odd integer, then n is also an odd integer. (Think about this before writing down your proof, you may even want to work it out on the back of another page, then write your final proof clearly here. You don't have to name any rules of inference like you did in the previous problem.)

b. [5] Was your proof in part **a** a direct proof, an indirect proof (by contraposition), or a proof by contradiction?