CLARK UNIVERSITY

Your name: ___________________________

Math 114 Discrete Mathematics

Second Midterm

April 2018

You may use a calculator and one sheet of prepared notes during this test. Show all your work for credit. Points for each problem are in square brackets.

1. [15; 5 points each part] Give a big-$\mathcal{O}$ estimate for each of these functions. For the function g in your estimate that $f(x)$ is $\mathcal{O}(g)$, use a simple function g of smallest order. (You don't need to prove that your answer is correct.)

a. $\dfrac{x^4 + \log x}{2x^2 + 3x + 1}$

b. $(x \log x + x^2)(x^3 + 2)$

c. $x^x + 2^x + x!$

2. [15] Use the definition of divisibility to prove that if a divides b, and c divides d, then ac divides bd. Assume that a, b, c, and d are all positive integers.

3. [10; 5 points each part] Evaluate the following quantities.

a. $100 \bmod 7$

b. $-3 \bmod 5$

4. [10] Use the Euclidean algorithm to find the greatest common divisor of 1039 and 323. (Show at least a couple of the intermediate steps.)

5. [12; 6 points each part] For this problem, you may leave your answers as algebraic expressions.

a. How many strings are there of lowercase letters of length four or less?

b. How many strings are there of lowercase letters of length four or less that have the letter x in them?

6. [20] On mathematical induction.

In this problem, you'll prove that $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$ for positive integers n.

a. [2] Prove the base case.

b. [2] State the inductive hypothesis when $n = k$.

c. [2] State the inductive conclusion when $n = k + 1$.

e. [14] Prove the inductive step that $n = k$ implies $n = k + 1$.

7. [20; 5 points each part] For this problem, you may leave your answers as expressions in terms of factorials and binomial coefficients.

A coin is flipped eight times where each flip comes up either heads or tails. How many possible outcomes

a. are there in total?

b. contain exactly three heads?

c. contain at least three heads?

d. contain the same number of heads and tails?

#1.[15]	
#2.[15]	
#3.[10]	
#4.[10]	
#5.[12]	
#5.[20]	
#5.[20]	
Total	