# Section 1.3, selected answers
## Math 114 Discrete Mathematics
D Joyce, Spring 2018

**2.** Let $P(x)$ be the statement "the word $x$ contains the letter $a$." What are the truth values?

**a.** $P$(orange). That's the statement "the word orange contains the letter $a$." A logician might quibble that orange is not a word but a fruit, so the statement is false; the logician would say 'orange' is the word. Let's not worry about that. Since the word 'orange' has an 'a' as its third letter, the statement is true.

Parts **b** and **c** are false since the words 'lemon' and 'true' don't have 'a's in them, but **d** is true since the word 'false' does have an 'a'.

**6.** Let $N(X)$ be "$x$ has visited North Dakota," where the domain includes all the students in your school. Express these quantifications in English.

**a.** $\exists x\, N(x)$. Someone in the school has visited North Dakota.

**b.** $\forall x\, N(x)$. Everyone in the school has visited North Dakota.

**c.** $\neg\exists x\, N(x)$. No one in the school has visisted North Dakota.

**d.** $\exists x\, \neg N(x)$. There is someone in the school who hasn't visited North Dakota.

**e.** $\neg\forall x\, N(x)$. Not everyone in the school has visited north Dakota. (Note that this is logically equivalent to **d** but it's expressed differently.)

**f.** $\forall x\, \neg N(X)$. Everyone in the school has not visited North Dakota. (Note that this is logically equivalent to **c** but it's expressed differently.)

**10.** Let $C(x)$ be the statement "$x$ has a cat," let $D(x)$ be the statement "$x$ has a dog," let $F(x)$ be the statement "$x$ has a ferret." Let the domain consist of all students in your class. Express the following statements in terms of $C, D, F$, quantifiers, and logical connectives.

Let's write $Cx$ rather than $C(x)$ so that we don't have as many parentheses.

**a.** A student in your class has a cat, a dog, and a ferret. $\exists x\,(Cx \wedge Dx \wedge Fx)$.

**b.** All students in your class have a cat, a dog, and a ferret. $\forall x\,(Cx \wedge Dx \wedge Fx)$.

**c.** Some student in your class has a cat and a ferret, but not a dog. $\exists x\,(Cx \wedge Fx \wedge \neg Dx)$.

**d.** No student in your class has a cat, a dog, and a ferret. $\neg\exists x\,(Cx \wedge Dx \wedge Fx)$.

**e.** For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has one of these animals as a pet. One expression that works is $\exists x\, Cx \wedge \exists y\, Dy \wedge \exists z\, Fz$. When you write it that way, it's easy to see that there can be three different pet owners in the class. But usually you'll see $\exists x\, Cx \wedge \exists x\, Dx \wedge \exists x\, Fx$. The three $x$s can refer to different people.

**15.** Determine the truth value. The universe of discourse is the set of integers. Think of these as questions.

**a.** $\forall n\,(n^2 \geq 0)$. Is the square of any integer greater or equal to 0? Yes. No square is negative.

**b.** $\exists n\,(n^2 = 2)$. Is there an integer whose square is 2? No. The solutions $n = \pm\sqrt{2}$ are not integers.

**c.** $\forall n\,(n^2 \geq n)$. Is the square of every integer greater than or equal to the integer itself? It is if the number is an integer.

**d.** $\exists n\,(n^2 < 0)$. Is there any integer whose square is negative? No. See part a above.

**16.** Determine the truth value . The universe of discourse is the set of real numbers. Think of these as questions.

**a.** $\exists x\,(x^2 = 2)$. Is there a number whose square is 2? Yes, in fact, two of them $\pm\sqrt{2}$.

**b.** $\exists x\,(x^2 = -1)$. Are there real numbers whose square is $-1$? No, only complex ones, $\pm i$.

**c.** $\forall x\,(x^2 + 2 \geq 1)$. If you take any number $x$, square it, and add 2, do you get a number that is at least 1? Yes, since after you square it, it's at least 0, so adding 2 makes it at least 2, which is bigger than 1.

**d.** $\forall x\,(x^2 \neq x)$. Can you ever square a number and get that very number? Yes, that happens when you start with either 0 or 1. So the statement $\forall x\,(x^2 \neq x)$ is false.

**36.** Find a counterexample, if possible, to these universally quantified statements, where the universe of discourse is the set of all real numbers.

**a.** $\forall x\,(x^2 \neq x)$. Are there any real numbers whose squares are equal to themselves? Yes, both 0 and 1. So there are counterexamples to this statement.

**b.** $\forall x\,(x^2 \neq 2)$. Counterexamples are $\pm\sqrt{2}$.

**c.** $\forall x\,(|x| > 0)$. Is the absolute value of every real number a positive number? Almost every real number, but $0^2$ is not positive. So a counterexample for part c is $x = 0$.

**52.** If the universe of discourse is the set of integers, which of the following are true?

**a.** $\exists! x\,(x > 1)$. False. There's more than one integer larger than 1.

**b.** $\exists! x\,(x^2 = 1)$. False. There are two square roots of 1, namely $+1$ and $-1$.

**c.** $\exists! x\,(x + 3 = 2x)$. True. The unique solution is $x = 3$.

**d.** $\exists! x\,(x = x + 1)$. False. There are no solutions to this equation.

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