Section 1.5, selected answers
Math 114 Discrete Mathematics
D Joyce, Spring 2018

4. What rule of inference is used in each of the following arguments.

a. Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials. $p \wedge q \to p$, called simplification in table 1 on page 66.

b. It's either hotter than 100 degrees today or the pollution is dangerous. It's less than 100 degrees outside today. Therefore, the pollution is dangerous. $(p \vee q) \wedge \neg p \to q$. Disjunctive syllogism.

c. Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard. $p \wedge (p \to q) \to q$. Modus ponens.

d. Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum. $p \to p \vee q$. Addition. (Editorial comment: addition looks like it could be used to make derogatory remarks about someone yet be logically correct.)

e. If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material. $(p \to q) \wedge (q \to r) \to (q \to r)$. Hypothetical syllogism.

8. What rules of inference are used here? "No man is an island. Manhattan is an island. Therefore, Manhattan is not a man."

Here's the form of the argument. $\neg\exists x\ (Mx \wedge Ix)$. $I(\texttt{Manhattan})$. Therefore $\neg M(\texttt{Manhattan})$.

The first statement is equivalent to $\forall x\ (\neg Mx \vee \neg Ix)$. Using universal instantiation, we get $\neg M(\texttt{Manhattan}) \vee \neg I(\texttt{Manhattan})$. Then, using disjunctive syllogism on that and the statement $I(\texttt{Manhattan})$, we conclude $\neg M(\texttt{Manhattan})$.

9. Draw relevant conclusions and state what rules of inference you use.

a. Lines 1 through 4 are given information. There are many ways to proceed. One is listed below.

1. "If I take the day off, it either rains or snows." $\forall x\ (Ox \to Rx \vee Sx)$.
2. "I took Tuesday off or I took Thursday off." $O(\texttt{Tuesday}) \vee O(\texttt{Thursday})$.
3. "It was sunny on Tuesday." $N(\texttt{Tuesday})$.
4. "It did not snow on Thursday." $\neg S(\texttt{Thursday})$.
5. "It rained or snowed on Tuesday or it rained or snowed on Thursday." $R(\texttt{Tuesday}) \vee S(\texttt{Tuesday}) \vee R(\texttt{Thursday}) \vee S(\texttt{Thursday})$. From 1 and 2 by a rule not mentioned in Table 1 on page 66. It's a version of modus ponens that applies to disjunctions, a kind of case argument. From $P \vee Q$ and $P \to R$ and $Q \to S$, you may conclude $R \vee S$.
6. "It did not rain or snow on Tuesday." $\neg(R(\texttt{Tuesday}) \vee S(\texttt{Tuesday}))$. From 3 and common knowledge. This is not actually a valid conclusion, since in a valid argument requires all hypotheses to be clearly stated. To get this conclusion validly, we would need the statement "If it rains or snows, then it is not sunny." then use modus tollens.
7. "I didn't take Tuesday off." $\neg O(\texttt{Tuesday})$. From 1 and 6 by modus tollens.
8. "I took Thursday off." $O(\texttt{Thursday})$. Disjunctive syllogism from 2 and 7.
9. "If I took Thursday day off, it either rained or snowed." $(O(\texttt{Thursday}) \to R(\texttt{Thursday}) \vee S(\texttt{Thursday}))$. Universal instantiation from 1.
10. "It either rained on Thursday or it snowed on Thursday." $R(\texttt{Thursday}) \vee S(\texttt{Thursday}))$. Modus ponens from 8 and 9.
11. "It rained on Thursday!" $R(\texttt{Thursday}))$. From 4 and 10 by disjunctive syllogism.

b.

1. "If I eat spicy foods, then I have strange dreams." $F \to D$.
2. "I have strange dreams if there is thunder while I sleep." $T \to D$.
3. "I did not have strange dreams." $\neg D$.
4. "I didn't eat spicy foods." $\neg F$. Modus tollens from 1 and 3.
5. "There wasn't any thunder while I slept." $\neg T$. Modus tollens from 2 and 3.
6. "Not only didn't I eat spicy foods, but it also didn't thunder while I slept." $\neg F \wedge \neg T$. Conjunction from 4 and 5.

c.

1. "I am either clever or lucky."
2. "I am not lucky."
3. "If I am lucky, then I will win the lottery."
4. "I am clever." Disjunctive syllogism, 1, 2.

Unfortunately, you can't make any conclusion about winning the lottery.

10. Draw relevant conclusions and state what rules of inference you use.

a.

1. "If I play hockey, then I am sore the next day." $H \to S$.
2. "I use the whirlpool if I am sore." $S \to W$.
3. "I did not use the whirlpool." $\neg W$.
4. "I was not sore." $\neg S$. Modus tollens from 2 and 3.
5. "I did not play hockey the previous day." $\neg H$. Modus tollens from 1 and 4.

b.

1. "If I work, it's either sunny or partly sunny." This is actually a quantified statement. It says that for each day x, if I work on x, then it's either sunny on x or it's partly sunny on x. We can write this symbolically as $\forall x\,(Wx \to Sx \vee Px)$.

2. "I worked last Monday or I worked last Friday." $W(\text{Monday}) \vee W(\text{Friday})$.

3. "It was not sunny on Tuesday." $\neg S(\text{Tuesday})$.

4. "It was not partly sunny on Friday." $\neg P(\text{Friday})$.

There are various conclusions you can make from these statements. Statements 1 and 2 say something about working, so you can make the following conclusion.

5. $S(\text{Monday}) \vee P(\text{Monday}) \vee S(\text{Friday}) \vee P(\text{Friday})$. From 1 and 2 by Universal Instantiation & various propositional rules.

Since 1 and 3 you can make some conclusions, too.

6. "If I worked on Tuesday, then it was partly sunny then." $W(\text{Tuesday}) \to P(\text{Tuesday})$. Use Universal Instantiation to apply 1 to the particular day Tuesday, then various propositional rules to derive 6 from 3.

Similarly from 1 and 4 you can conclude

7. "If I worked on Friday, then it was sunny then." $W(\text{Friday}) \to S(\text{Friday})$.

If you like, you can combine information from 2, 5, and 7 to make various conclusions about the weather, but none of them say very much.

c.

1. "All insects have six legs." $\forall x\,(Ix \to Sx)$.

2. "Dragonflies are insects." $I(\text{Dragonflies})$.

3. "Spiders don't have six legs." $\neg S(\text{Spiders})$.

4. "Spiders eat dragonflies." $E(\text{Spiders}, \text{Dragonflies})$.

From 1 and 2 you can conclude dragonflies have six legs. Here's how.

5. $I(\text{Dragonflies}) \to S(\text{Dragonflies})$. From 1 via Universal Instantiation.

6. $S(\text{Dragonflies})$. From 2 and 5 by Modus Ponens.

Similarly, from 1 and 2 using Universal Instantiation and Modus Tollens you can derive

7. "Spiders aren't insects." $\neg I(\text{Spiders})$.

Now with 4, you can derive that some noninsects eat insects. Here's how.

8. $E(\text{Spiders}, \text{Dragonflies}) \wedge S(\text{Dragonflies}) \wedge \neg I(\text{Spiders})$. From 4, 6, and 7 by Conjunction.

9. "Some noninsects eat insects." $\exists x\,\exists y\,(Exy \wedge Sy \wedge \neg Ix)$. From 8 by two applications of Existential Generalization.

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