Section 1.6, selected answers
Math 114 Discrete Mathematics
D Joyce, Spring 2018

6. Use a direct proof to show that the product of two odd integers is odd.

Use the definition that n is an odd when $\exists k, n = 2k + 1$. There are various wordings you can use in your proof, but they'd all have the same structure.

Let m and n be odd integers. Then $\exists k, n = 2k + 1$ and $\exists j, m = 2j+1$. Then the product mn equals $(2k+1)(2j+1)$ which can be written as $2(2jk+k+j)+1$ which shows that $\exists i, mn = 2i + 1$, namely $i = 2jk + k + j$. Q.E.D.

10. Use a direct proof to show that the product of two rational numbers is also rational.

Suppose that x and y are both rational. Then there are integers m and n so that $x = m/n$, and there are integers p and q so that $y = p/q$. Multiplying, we can conclude $xy = (mp)/(nq)$. That exhibits xy as a quotient of integers. Therefore the product xy is also rational. Q.E.D.

11. Prove or disprove that the product of two irrational numbers is irrational.

This is false, and all you have to do is exhibit a counterexample.

Since the two irrational numbers $\sqrt{2}$ and $\sqrt{2}$ have the rational number 2 as their product, it is not the case that the product of two irrational numbers is always irrational.

39. Prove that at least one of the real numbers $a_1, a_2, \ldots, a_n$ is greater than or equal to the average of these numbers. What kind of proof did you use?

Almost surely the proof you came up with is a nonconstructive existential proof in the form of an indirect proof.

Suppose every a_i is less than the average

$$\overline{a} = (a_1 + a_2 + \cdots + a_n)/n.$$

That is, $a_1 < \overline{a}$, $a_2 < \overline{a}$, $\ldots$, $a_n < \overline{a}$. Add these n inequalities together to get $a_1 + a_2 + \cdots + a_n < n\overline{a}$. But $n\overline{a} = a_1 + a_2 + \cdots + a_n$. But, as Euclid would say, it is absurd that a number be both less than and equal to itself. Therefore, not every a_i is less than the average. Thus, some a_i is greater than or equal to the average. Q.E.D.

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