Section 3.4, selected answers
Math 114 Discrete Mathematics
D Joyce, Spring 2018

2. Show that if a is an integer other than 0, then

a. 1 divides a. But of course, $1 \cdot a = a$, so 1 divides a.

b. a divides 0. For sure, since $a \cdot 0 = 0$.

6. Prove $a|c$ and $b|d$ implies $ab|cd$.

There's not much choice for this proof. A direct proof that relies only on the definition and a little algebra is enough.

Proof: Suppose $a|c$ and $b|d$. Then there are numbers e and f such that $ae = c$ and $bf = d$. Therefore $aebf = cd$. But $(ab)(ef) = cd$, therefore $ab|cd$. Q.E.D.

11. Let m be a positive integer. Show that $a \bmod m = b \bmod m$ if $a \equiv b \pmod m$.

This is a more complicated proof, and yours may not look much like theone I came up with.

Proof: Suppose $a \equiv b \pmod m$. Then $m|(a-b)$. Let $a \bmod m$ be r, and let $b \bmod m$ be s. That means

$$a = mq + r$$

where $0 \leq r < m$ and q is some integer; also

$$b = mt + s$$

where $0 \leq s < m$ and t is some integer. Subtracting we find that

$$a - b = m(q - t) + (r - s).$$

But m divides $a - b$, so m divides $m(q-t) + (r-s)$, and since m divides $m(q-t)$, therefore m also divides $r - s$. But $r - s$ is a number greater than $-m$ and less than m, and the only number in that range that m divides is 0. Hence, $r - s = 0$, so $r = s$. Therefore $a \bmod m = b \bmod m$. Q.E.D.

16. Evaluate these quantities.

a. $-17 \bmod 2$. Since -17 is negative, you have to be a little careful. Any integer modulo 2 has to be either 0 or 1, and since -17 is odd, therefore $-17 \bmod 2 = 1$.

b. $144 \bmod 7$. When you divide 7 into 144, you get a remainder of 4, so $144 \bmod 7 = 4$.

c. $-101 \bmod 13$. Any integer modulo 13 has to be some integer from 0 through 12, inclusive. Now, $101 \bmod 13 = 10$, so $-101 \bmod 13$ has to be congruent to -10 modulo 13, but some integer between 0 and 12. Adding 13 to -10 gives 3, so $-101 \bmod 13 = 3$. For the most part, we're not interested in negative numbers, but it's nice that the definition covers them.

d. $199 \bmod 19$. When you divide 19 into 199, you get a remainder of 9, so $199 \bmod 19 = 9$.

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