2. Express the greatest common divisor of each of these pairs of integers as a linear combination of these integers.

a. 9, 11. The gcd is 1. We need to find 1 as a linear combination of 9 and 11. These are small enough numbers so we can do it by searching. We need to find a multiple of 9 that is one more or less than a multiple of 11. The multiples of 9 are 9, 18, 27, 36, 45. Stop. 45 is 1 more than 44. That is 9 · 5 = 1 + 4 · 11. Thus,
   \[ 1 = (-5) \cdot 9 + 4 \cdot 11 \]
expresses the gcd of 9 and 11, namely, 1, as a linear combination of 9 and 11.

b. 33, 44. The gcd is 11, but 11 is 44 − 33, thus
   \[ 11 = (-1) \cdot 33 + 1 \cdot 44 \]
expresses 11 as a linear combination of 33 and 44.

c. 35, 78. The numbers are getting larger. A search would work, but the general method is probably just as fast. We’ll use the Euclidean algorithm keeping track of our computations and build the answer from that.
   \[
   \begin{align*}
   78 &= 2 \cdot 35 + 8 \\
   35 &= 4 \cdot 8 + 3 \\
   8 &= 2 \cdot 3 + 2 \\
   3 &= 1 \cdot 2 + 1
   \end{align*}
   \]
Now, build up the answer starting with the last equation
\[
1 = 3 \cdot 2 \\
 = 3 \cdot (8 - 2 \cdot 3) \\
 = 3 \cdot 3 \cdot 8 \\
 = 3 \cdot 3 \cdot (35 - 4 \cdot 8) - 8 \\
 = 3 \cdot 35 - 13 \cdot 8 \\
 = 3 \cdot 35 - 13 \cdot (78 - 2 \cdot 35) \\
 = 29 \cdot 35 - 13 \cdot 78
\]

6. Find an inverse of 2 modulo 17.
   
   Which multiple of 2 is one more than 17? 2 times 9 equals 18. Thus, 2 · 9 ≡ 1 (mod 17), so 8 is an inverse of 2 modulo 17.

18. Find all solutions to the system of congruences
   \[
   \begin{align*}
   x &\equiv 2 \pmod{3} \\
   x &\equiv 1 \pmod{4} \\
   x &\equiv 3 \pmod{5}
   \end{align*}
   \]