

Math 114 Discrete Mathematics
Section 5.2, selected answers
D Joyce, Spring 2018

1. Show that in any set of six classes there must be two that meet on the same day, assuming that no classes are held on weekends.

Since there are only five weekdays, if there are six classes, by the pigeonhole principle, two have to meet on the same day.

2. Show that if there are 30 students in a class, then at least 2 have last names that begin with the same letter.

Since $30 > 26$, by the pigeonhole principle, two have last names that begin with the same letter.

6. Let d be a positive integer. Show that among any group of $d+1$ (not necessarily consecutive) integers there are two with exactly the same remainder when they are divided by d .

Since there are only d possible remainders modulo d (namely, $0, 1, \dots, d-1$), therefore at least two of the $d+1$ remainders would be equal.

20. Find an increasing subsequence of maximal length and a decreasing subsequence of maximal length in the sequence

22, 5, 7, 2, 23, 10, 15, 21, 3, 17.

There are algorithms to do this, but it appears that the job here is to be done by searching for the answer. It looks like the longest increasing subsequence has length five, either 5, 7, 10, 15, 21, or 5, 7, 10, 15, 17. There are lots of decreasing subsequences of length three, for instance, 22, 7, 2, but it doesn't appear that there are any of length four.

21. Construct a sequence of 16 positive integers that has no increasing or decreasing subsequence of 5 terms.

There are many, but here is one:

4, 3, 2, 1, 8, 7, 6, 5, 12, 11, 10, 9, 16, 15, 14, 13.

The pattern is evident—four decreasing numbers, then four larger decreasing numbers, etc.

30. Show that if there are 100,000,000 wage earners in the United States who earn less than 1,000,000 dollars, then there are two who earned exactly the same amount of money, to the penny, last year.

The possible earnings go from \$0.01 to \$999,999.99. That's $10^8 - 1$ different values. (Apparently, \$0.00 is excluded in this exercise since someone is not a "wage earner" unless a positive wage is earned.) But there are 10^8 wage earners, so, by the pigeon hole principle, two of the same wage earners have to earn the same wage.

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