Math 114 Discrete Mathematics
Section 8.6, selected answers
D Joyce, Spring 2018

7. Determine whether the relations represented by these 0-1 matrices are partial orders.

They're all reflexive since they all have 1's down the main diagonals. They're all antisymmetric since whenever the ijth entry is 1, then the jith entry is 0. Matrix **a** doesn't describe a transitive relation since the 2-1 entry is 1, and the 1-3 entry is 1, but the 2-3 entry is 0. Matrix **b** is transitive, and so is matrix **c**. So they both represent partial orders.

9-11. Determine whether the relation with the directed graph is a partial order.

They're all reflexive since there's a loop at every vertex. They're antisymmetric since whenever an arrow goes from one vertex to another, there's never an arrow back from the other to the first. Both 10 and 11 are transitive since for any consecutive pair of arrows, the "composition" arrow is there, too. But 9 is not transitive since there are arrows from a to b and from b to c, but no arrow from a to c. Thus, 10 and 11 are partial orders, but 9 is not.

15. Find two incomparable elements in the following posets.

a. The power set of $\{0, 1, 2\}$ under subset inclusion.

Find two subsets neither of which is contained in the other. For example $\{1, 2\}$ and $\{2, 3\}$.

b. The set $\{1, 2, 4, 6, 8\}$ under divisibility.

Find two numbers in the set neither of which evenly divides the other. For example, 6 and 8.

16. Let S be the set $\{1, 2, 3, 4\}$. With respect to the lexicographic order based on the usual "less than" relation,

a. find all pairs in $S \times S$ less than $(2, 3)$.

Either the first coordinate is 1, or the first coordinate is 2 but the second coordinate is less than 3. So the pairs are $(1, 1), (1, 2), (1, 3), (1, 4), (2, 1)$, and $(2, 2)$.

b. find all pairs in $S \times S$ greater than $(3, 1)$.

They are $(3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3)$, and $(4, 4)$.

c. draw the Hasse diagram of the whole poset $S \times S$ with the lexicographic order.

It's just a vertical line with 16 points labelled from the bottom to the top
$(1, 1)$, $(1, 2)$, $(1, 3)$, $(1, 4)$, $(2, 1)$, $(2, 2)$, $(2, 3)$, $(2, 4)$, $(3, 1)$, $(3, 2)$, $(3, 3)$, $(3, 4)$, $(4, 1)$, $(4, 2)$, $(4, 3)$, $(4, 4)$.

22. Draw Hasse diagrams for divisibility on the sets.

a. $\{1, 2, 3, 4, 5, 6\}$.

1 at the bottom; 2, 3, and 5 above 1; 4 above 2; and 6 above both 2 and 3.

b. $\{3, 5, 7, 11, 13, 16, 17\}$.

All the numbers are incomparable, so the diagram just has seven vertices labelled with the numbers and no lines between them.

c. $\{2, 3, 5, 10, 11, 15, 25\}$.

Since 2, 3, 5, and 11 are incomparable, put them on the bottom. Put 10 above 2 and 5; 15 above 2 and 5; and 25 above 5.

d. $\{1, 3, 9, 27, 81, 243\}$.

It's a linear Hasse diagram since each number divides the next.

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