Quiz
Math 114 Discrete Mathematics
D Joyce, Feb 2018

Scale. 9–10 A, 7–8 B, 5–6 C. Median 8.5.

1. [5 points] Let $Q(x, y)$ be the statement "student x has been a contestant on quiz show y." Express each of these sentences in terms of $Q(x, y)$, quantifiers, and logical connectives, where the domain for x consists of students and the domain for y consists of quiz shows.

a. There is a student who has been a contestant on a quiz show.

$$\exists x, \exists y, Q(x, y)$$

b. No student has ever been a contestant on a quiz show.

$$\neg\exists x, \exists y, Q(x, y)$$

There are other ways to express this including $\forall x, \neg\exists y, Q(x, y)$, and $\forall x, \forall y, \neg Q(x, y)$.

c. There is a student who has been a contestant on both the quiz show Jeopardy and the quiz show Wheel of Fortune.

$$\exists x\, (Q(x, \text{Jeopardy}) \land Q(x, \text{WheelofFortune}))$$

2. [5 points] Use rules of inference to show that the three hypotheses

(i) "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on."

(ii) "If the sailing race is held, then the trophy will be awarded."

(iii) "The trophy was not awarded."

imply the conclusion

(iv) "It rained."

a. Let R stand for it rains, F for it's foggy, S for the sailing race being held, L for the lifesaving demonstration, and T for the trophy being awarded. Write the four statements (i) through (iv) in terms of R, F, S, L, and T.

(i). $(\neg R \lor \neg F) \to (S \land L)$.

(ii). $S \to T$.

(iii). $\neg T$.

(iv). R.

b. Show that (iv) follows from (i), (ii), and (iii). Break down the steps in the argument into the smallest pieces you can. You don't have to explicitly name the rules such that you use, such as "modus ponens", but each step needs to be as simple as you can make it.

There are various proofs. Here are two. I'll include the justifications although they're not required for your answers.

First proof: The contrapositive of (i) says $\neg(S \land L) \to \neg(\neg R \lor \neg F)$. Using De Morgan's laws, that can be rewritten as $\neg S \lor \neg L \to R \land F$.

From (ii) and (iii) by modus tollens, therefore $\neg S$. By the addition rule, therefore $\neg S \lor \neg L$.

From $\neg S \lor \neg L \to R \land F$ and $\neg S \lor \neg L$, we get $R \land F$ by modus ponens. Then by simplification, R. Q.E.D.

Second proof: From (ii) and (iii) by modus tollens, therefore $\neg S$.

Since $\neg S$, therefore $\neg(S \land L)$. This is actually the contrapositive of the rule of simplification $S \land L \to S$.

From that statement and (ii), therefore, by modus tollens, $\neg(\neg R \lor \neg F)$.

That's equivalent to $R \land F$ by De Morgan's laws.

Therefore, R, by simplification again. Q.E.D.