CLARK UNIVERSITY 1887

Practice Integration
Math 120 Calculus I

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This first set of indefinite integrals, that is, antiderivatives, only depends on a few principles of integration, the first being that integration is inverse to differentiation. Besides that, a few rules can be identified: a constant rule, a power rule, linearity, and a limited few rules for trigonometric, logarithmic, and exponential functions.

$$\int k\,dx = kx + C, \qquad \text{where } k \text{ is a constant}$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C, \qquad \text{if } n \neq -1$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int kf(x)\,dx = k\int f(x)\,dx$$

$$\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \frac{1}{1+x^2}\,dx = \arctan x + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$$

We'll add more rules later, but there are plenty here to get acquainted with.

Here's a list of practice exercises. There's a hint for each one as well as an answer with intermediate steps.

1. $\int (x^4 - x^3 + x^2)\,dx$. Hint. Answer.

2. $\int (5t^8 - 2t^4 + t + 3)\,dt$. Hint. Answer.

3. $\int (7u^{3/2} + 2u^{1/2})\,du$. Hint. Answer.

4. $\int (3x^{-2} - 4x^{-3})\,dx$. Hint. Answer.

5. $\int \frac{3}{x}\,dx$. Hint. Answer.

6. $\int \left(\frac{4}{3t^2} + \frac{7}{2t}\right)dt$. Hint. Answer.

7. $\int \left(5\sqrt{y} - \frac{3}{\sqrt{y}}\right)dy$. Hint. Answer.

8. $\int \frac{3x^2 + 4x + 1}{2x}\,dx$. Hint. Answer.

9. $\int (2\sin\theta + 3\cos\theta)\,d\theta$. Hint. Answer.

10. $\int (5e^x - e)\,dx$. Hint. Answer.

11. $\int \frac{4}{1+t^2}\,dt$. Hint. Answer.

12. $\int (e^{x+3} + e^{x-3})\,dx$. Hint. Answer.

13. $\int \frac{7}{\sqrt{1-u^2}}\,du$. Hint. Answer.

14. $\int \left(r^2 - 2r + \frac{1}{r}\right) dr$. Hint. Answer.

15. $\int \frac{4\sin x}{3\tan x}\, dx$. Hint. Answer.

16. $\int (7\cos x + 4e^x)\, dx$. Hint. Answer.

17. $\int \sqrt[3]{7v}\, dv$. Hint. Answer.

18. $\int \frac{4}{\sqrt{5t}}\, dt$. Hint. Answer.

19. $\int \frac{1}{3x^2+3}\, dx$. Hint. Answer.

20. $\int \frac{x^4 - 6x^3 + e^x\sqrt{x}}{\sqrt{x}}\, dx$. Hint. Answer.

1. Hint. $\int (x^4 - x^3 + x^2)\, dx$.

Integrate each term using the power rule,

$$\int x^n\, dx = \frac{1}{n+1}x^{n+1} + C.$$

So to integrate x^n, increase the power by 1, then divide by the new power. Answer.

2. Hint. $\int (5t^8 - 2t^4 + t + 3)\, dt$.

Remember that the integral of a constant is the constant times the integral. Another way to say that is that you can pass a constant through the integral sign. For instance,

$$\int 5t^8\, dt = 5\int t^8\, dt$$

Integrating polynomials is fairly easy, and you'll get the hang of it after doing just a couple of them. Answer.

3. Hint. $\int (7u^{3/2} + 2u^{1/2})\, du$.

You can use the power rule for other powers besides integers. For instance,

$$\int u^{3/2}\, du = \tfrac{2}{5}u^{5/2} + C$$

Answer.

4. Hint. $\int (3x^{-2} - 4x^{-3})\, dx$

You can even use the power rule for negative exponents (except -1). For example,

$$\int x^{-3}\, dx = -\tfrac{1}{2}x^{-2} + C$$

Answer.

5. Hint. $\int \frac{3}{x}\, dx$

This is $3x^{-1}$ and the general power rule doesn't apply. But you can use

$$\int \frac{1}{x}\, dx = \ln|x| + C.$$

Answer.

6. Hint. $\int \left(\frac{4}{3t^2} + \frac{7}{2t}\right) dt$

Treat the first term as $\frac{4}{3}t^{-2}$ and the second term as $\frac{7}{2}t^{-1}$. Answer.

7. Hint. $\int \left(5\sqrt{y} - \frac{3}{\sqrt{y}}\right) dy$

It's usually easier to turn those square roots into fractional powers. So, for instance, $\frac{1}{\sqrt{y}}$ is $y^{-1/2}$. Answer.

8. Hint. $\displaystyle\int \frac{3x^2+4x+1}{2x}\,dx$

Use some algebra to simplify the integrand, that is, divide by $2x$ before integrating. Answer.

9. Hint. $\displaystyle\int (2\sin\theta + 3\cos\theta)\,d\theta$

Getting the $\pm$ signs right when integrating sines and cosines takes practice. Answer.

10. Hint. $\displaystyle\int (5e^x - e)\,dx$

Just as the derivative of e^x is e^x, so the integral of e^x is e^x. Note that the $-e$ in the integrand is a constant. Answer.

11. Hint. $\displaystyle\int \frac{4}{1+t^2}\,dt$

Remember that the derivative of $\arctan t$ is $\dfrac{1}{1+t^2}$. Answer.

12. Hint. $\displaystyle\int (e^{x+3} + e^{x-3})\,dx$

When working with exponential functions, remember to use the various rules of exponentiation. Here, the rules to use are $e^{a+b} = e^a e^b$ and $e^{a-b} = e^a/e^b$. Answer.

13. Hint. $\displaystyle\int \frac{7}{\sqrt{1-u^2}}\,du$

Remember that the derivative of $\arcsin u$ is $\dfrac{1}{\sqrt{1-u^2}}$ Answer.

14. Hint. $\displaystyle\int \left(r^2 - 2r + \frac{1}{r}\right) dr$

Use the power rule, but don't forget the integral of $1/r$ is $\ln|r| + C$. Answer.

15. Hint. $\displaystyle\int \frac{4\sin x}{3\tan x}\,dx$

You'll need to use trig identities to simplify this. Answer.

16. Hint. $\displaystyle\int (7\cos x + 4e^x)\,dx$

Just more practice with trig and exponential functions. Answer.

17. Hint. $\displaystyle\int \sqrt[3]{7v}\,dv$

You can write $\sqrt[3]{7v}$ as $\sqrt[3]{7}\,\sqrt[3]{v}$. And remember you can write $\sqrt[3]{v}$ as $v^{1/3}$. Answer.

18. Hint. $\displaystyle\int \frac{4}{\sqrt{5t}}\,dt$

Use algebra to write this in a form that's easier to integrate. Remember that $1/\sqrt{t}$ is $t^{-1/2}$. Answer.

19. Hint. $\displaystyle\int \frac{1}{3x^2+3}\,dx$

You can factor out a 3 from the denominator to put it in a form you can integrate. Answer.

20. Hint. $\displaystyle\int \frac{x^4 - 6x^3 + e^x\sqrt{x}}{\sqrt{x}}\,dx$

Divide through by $\sqrt{x}$ before integrating. Alternatively, write the integrand as

$$x^{-1/2}(x^4 - 6x^3 + e^x x^{1/2})$$

and multiply. Answer.

1. Answer. $\displaystyle\int (x^4 - x^3 + x^2)\,dx.$

The integral is $\frac{1}{5}x^5 - \frac{1}{4}x^4 + \frac{1}{3}x^3 + C$.

Whenever you're working with indefinite integrals like this, be sure to write the $+C$. It signifies that you can add any constant to the antiderivative $F(x)$ to get another one, $F(x) + C$.

When you're working with definite integrals with limits of integration, $\displaystyle\int_a^b$, the constant isn't needed since you'll be evaluating an antiderivative $F(x)$ at b and a to get a numerical answer $F(b) - F(a)$.

2. Answer. $\int (5t^8 - 2t^4 + t + 3)\, dt.$

The integral is $\frac{5}{9}t^9 - \frac{2}{5}t^5 + \frac{1}{2}t^2 + 3t + C.$

3. Answer. $\int (7u^{3/2} + 2u^{1/2})\, du.$

This integral evaluates as $\frac{14}{5}u^{5/2} + \frac{4}{3}u^{3/2} + C.$

4. Answer. $\int (3x^{-2} - 4x^{-3})\, dx.$

That equals $-3x^{-1} + 2x^{-2} + C$. If you prefer, you could write the answer as $-\dfrac{3}{x} + \dfrac{2}{x^2} + C$

5. Answer. $\int \dfrac{3}{x}\, dx$

That's $3\ln|x| + C$. The reason the absolute value sign is there is that when x is negative, the derivative of $\ln|x|$ is $1/x$, so by putting in the absolute value sign, you're covering that case, too.

6. Answer. $\int \left(\dfrac{4}{3t^2} + \dfrac{7}{2t}\right) dt.$

The integral of $\frac{4}{3}t^{-2} + \frac{7}{2}t^{-1}$ is $-\frac{4}{3}t^{-1} + \frac{7}{2}\ln|t| + C.$

7. Answer. $\int \left(5\sqrt{y} - \dfrac{3}{\sqrt{y}}\right) dy.$

The integral of $5y^{1/2} - 3y^{-1/2}$ is $\frac{10}{3}y^{3/2} - 6y^{1/2} + C.$ You could write that as $\frac{10}{3}y\sqrt{y} - 6\sqrt{y} + C$ if you prefer.

8. Answer. $\int \dfrac{3x^2 + 4x + 1}{2x}\, dx.$

The integral of $2x + 2 + \frac{1}{2}x^{-1}$ is

$$x^2 + 2x + \frac{1}{2}\ln|x| + C.$$

9. Answer. $\int (2\sin\theta + 3\cos\theta)\, d\theta.$

That's equal to $-2\cos\theta + 3\sin\theta + C.$

10. Answer. $\int (5e^x - e)\, dx$

That equals $5e^x - ex + C.$

11. Answer. $\int \dfrac{4}{1+t^2}\, dt.$

That evaluates as $4\arctan t + C$. Some people prefer to write $\arctan t$ as $\tan^{-1} t$.

12. Answer. $\int (e^{x+3} + e^{x-3})\, dx.$

The integrand is its own antiderivative, that is, the integral is equal to

$$e^{x+3} + e^{x-3} + C.$$

If you write the integrand as $e^x e^3 + e^x/e^3$, and note that e^3 is just a constant, you can see that it's its own antiderivative.

13. Answer. $\int \dfrac{7}{\sqrt{1-u^2}}\, du.$

The integral equals $7\arcsin u$.

14. Answer. $\int \left(r^2 - 2r + \dfrac{1}{r}\right) dr.$

The integral evaluates as

$$\tfrac{1}{3}r^3 - r^2 + \ln|r| + C.$$

15. Answer. $\int \dfrac{4\sin x}{3\tan x}\, dx$

The integrand simplifies to $\frac{4}{3}\cos x$. Therefore the integral is $\frac{4}{3}\sin x + C$.

16. Answer. $\int (7\cos x + 4e^x)\, dx.$

That's $7\sin x + 4e^x + C.$

17. Answer. $\int \sqrt[3]{7v}\, dv.$

Since you can rewrite the integrand as $\sqrt[3]{7}\, v^{1/3}$, therefore its integral is

$$\tfrac{3}{4}\sqrt[3]{7}\, v^{4/3} + C.$$

18. Answer. $\displaystyle\int \frac{4}{\sqrt{5t}}\,dt.$

The integral of $\dfrac{4}{\sqrt{5}}\,t^{-1/2}$ is equal to $\dfrac{8}{\sqrt{5}}\,t^{1/2}+C$. You could also write that as $8\sqrt{t/5}+C$.

19. Answer. $\displaystyle\int \frac{1}{3x^2+3}\,dx$

This integral equals $\frac{1}{3}\arctan x + C$.

20. Answer. $\displaystyle\int \frac{x^4-6x^3+e^x\sqrt{x}}{\sqrt{x}}\,dx.$

The integral can be rewritten as

$$\int (x^{7/2}-6x^{5/2}+e^x)\,dx$$

which equals $\frac{2}{9}x^{9/2}-\frac{12}{7}x^{7/2}+e^x+C$.

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