Math 128, Modern Geometry

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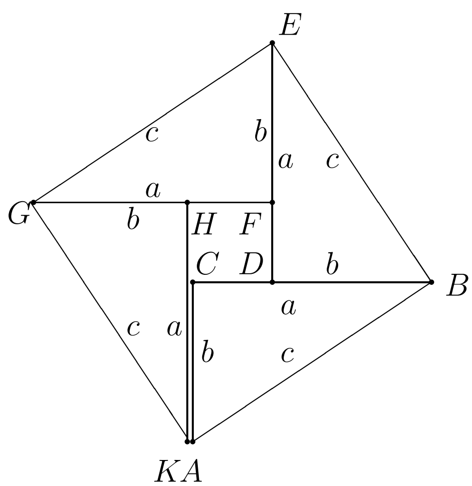
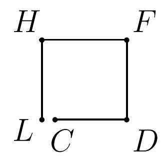
26 Sep 2005

At our last meeting we looked at a significant gap in our proof. It occurred after the construction that ended with this figure.



And we had next that "K coincides with A (we have to figure out why), and C lies on the line AH (again, we have to figure out why)." Well, we figured out why, or at least we found some principles that would help to fill in the gap. We added another point to the figure, L, on HJ, so that $LJ = b$. Then, ignoring most of the figure we concentrated on the center part of the figure.

This center part looks almost like a square, and we need it to be a square before going on in the proof. The four line segments, CD, DF, FH, and HL, are all equal since they're each the difference between the long and short sides of the four congruent triangles. Also, the three angles at D, F, and H are all right angles since they're supplementary to the right angles in the congruent trianges. But what we need to know is that the figure closes up so that the points L and C are the same.

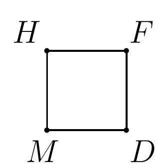


We came up with some arguments for that using the concept of parallelism. The two principles of parallelism that we came up with we'll take as axioms now, but we may be able to prove them later. The first principle is that two lines that are perpendicular to the same line are parallel. In our figure, HF and CD are both perpendicular to DF, so they're parallel. Also, LH and DF are perpendicular to HF, so they're parallel.

The second principle is that lines parallel to perpendicular lines are perpendicular to each other. Since $HF \| CD$, and $LH \| DF$, and $HF \perp DF$, therefore $HL \perp CD$.

Now we may conclude the figure closes up with the lines HL and DC meeting at some point M with a right angle at M. (Eventually, we'll show $L = C = M$, but right now we don't know that.)

We now have the figure



where all four angles are right, and $HF = FD$. Since the angles are all right, we can call the figure a *rectangle*. We want to know it's a square, but we only have two equal sides. That means we need another theorem, which we'll have to prove:

Theorem. If a rectangle has two equal adjacent sides, then the remaining two sides are also equal to them, so the rectangle is a square.

By now, we've identified several principles that we're using in the proof of the Pythagorean theorem. Some of the we expect to prove on even more basic principles, so they will end up being theorems. Some of these principles are definitions, like those for rectangles and squares mentioned above. And some, those that don't seem to be based on even simpler principles, will be are axioms.