Math 128, Modern Geometry

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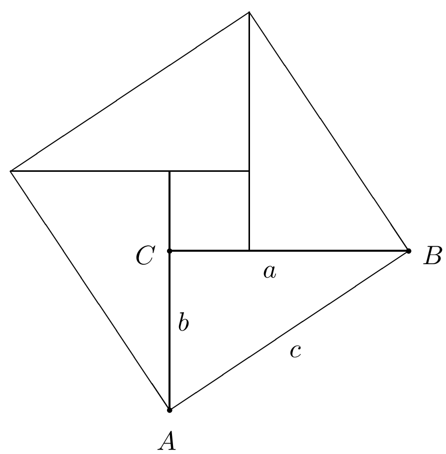
Last meeting we looked at several proofs of the Pythagorean theorem and selected one of them to look at in more detail. A couple of the proofs involved similar triangles, one involved trigonometry, and two were variants of the old Chinese proof.

We selected the following proof to consider in more detail.



Let ABC be a right triangle with right angle at C, and let a, b, and c be the sides opposite the vertices A, B, and C, respectively. Rotate the triagle 90°, 180°, and 270° to get three triangles congruent to the given triangle ABC. Place these with the hypotenuses facing outward so that the four hypotenuses form a c by c square. Inside there will be a small square hole whose side is $b - a$ (anyway that's what it is when $b > a$; but if $a > b$, then the side of the square hole will be $a - b$; and should it happen that $a = b$, then there's no hole at all). Then the large square is made out of four congruent triangles each with area $ab/2$ and the square hole of area $(b - a)^2$. Therefore,

$$c^2 = 4\,\frac{ab}{2} + (b-a)^2 = 2ab + b^2 - 2ab + a^2 = a^2 + b^2.$$

Thus, we've proved the Pythagorean theroem.

We'll analyze this proof in a little more detail to see what assumptions are needed for it to work. We'll divide the proof into two parts and then analyze each section. The first part is the construction of the figure. The second part begins with that constructed figure, then shows that $c^2 = a^2 + b^2$.