Math 128, Modern Geometry

D. Joyce, Clark University

28 Oct 2005

Last time. Finished our discussion of elliptic geometry. Began a discussion of absolute geometry, also called neutral geometry, the properties of geometry that are common to hyperbolic, Euclidean, and elliptic geometry.

For next time. Read chapter 13 about the projective plane.

Today. Finish discussion of absolute geometry.

Kepler's principle of continuity. Kepler saw certain figures as limits of other figures. For instance, if you look at circles centered at A with radius AB, and let A travel along a line $\perp$ to B going further and further away from B, then the circles get larger and larger and approach a straight line, the one $\perp$ to AB. Thus, a straight line is the limit of circles of growing radius.

We can go up a dimension and see the plane as the limit of spheres of growing radius. Each of these spheres is a model of elliptic geometry, but the scale of distance changes as the sphere enlarges. Only when the radius is 1 do we have the precise metric elliptic geometry we studied before. When we relax the definition of elliptic geometry so that the length of an elliptic straight line doesn't have to equal 2π (in the double elliptic plane, but π in the single elliptic plane), but it may equal $2\pi r$, then we include these spheres as models of the elliptic plane.

Curvature. The curvature k of a circle is the reciprocal of the radius r of that circle. Likewise, the curvature k of a sphere is the reciprocal of the radius r of that sphere. A large radius makes a small curvature. As the size of the radius approaches ∞, the curvature k approaches 0. The curvature of a flat plane is 0.

A surface that is not uniform can have different curvatures in different places. In a saddle, the curvature is negative. A surface with constant negative curvature is a model of a piece of the hyperbolic plane. The hyperbolic plane has constant negative curvature, in fact, if the scale is set as we did when we studied the hyperbolic plane, then that curvature is constantly -1. But if it's scaled differently, it can have a different constant negative curvature, so long as we relax the definition of hyperbolic geometry.

Just as the Euclidean plane is the limit of elliptic planes as their positive curvatures approach 0, the Euclidean plane is also the limit of hyperbolic planes as their negative curvatures approach 0.

Other plane geometries. We'll look at what we expect from a geometry and some of the other examples from the text. Our next subject will be one of these other geometries, namely, projective geometry.