Math 128, Modern Geometry

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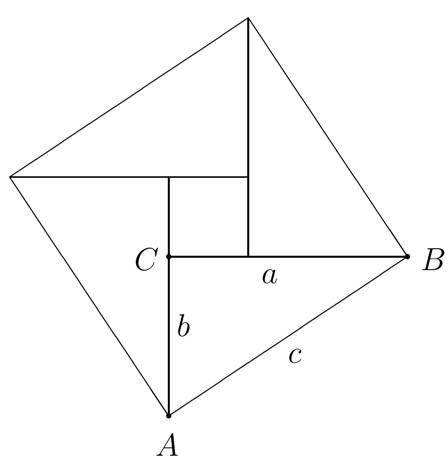
We started to examine the assumptions in this proof of the Pythagorean theroem. So far, we've only considered the first part of the proof which constructs the figure. The second part begins with that constructed figure, then shows that $c^2 = a^2 + b^2$.

Given information. Let ABC be a right triangle with right angle at C, and let a, b, and c be the sides opposite the vertices A, B, and C, respectively.



Part I. The construction of the figure. Rotate the triagle 90°, 180°, and 270° to get three triangles congruent to the given triangle ABC. Place these with the hypotenuses facing outward so that the four hypotenuses form a c by c square. Inside there will be a small square hole whose side is $b-a$ (anyway that's what it is when $b > a$; but if $a > b$, then the side of the square hole will be $a - b$; and should it happen that $a = b$, then there's no hole at all).

Part II. The derivation of the Pythagorean identity. Then the large $c \times c$ square is made out of four congruent triangles each with area $ab/2$ and the square hole of area $(b - a)^2$. Therefore,

$$c^2 = 4\,\frac{ab}{2} + (b-a)^2 = 2ab + b^2 - 2ab + a^2 = a^2 + b^2.$$

Here are some of the assumptions we've identified in part I:

1. Figures can be rotated. We rotate the triangle ABC by a quarter-turn three times.
2. Figures can be translated. We translate the three triangles to put them in particular positions.
3. This process results in forming a square out of the four hypotenuses.
4. Although we didn't mention it in class, analogous to the statement that a square results from the four hypotenuses, we could also mention that the inner figure is a square and the various sides of the triangles line up without any gap.

We also noted that a proper proof would consider separately the three cases $b > a$, $b = a$, and $b < a$. The first and third cases are the same except for the names of the sides. The second case where $b = a$ may have a simpler proof.

One question we had was whether to keep the language of rotations and make transformations part of the proof, or to drop rotations. If we drop rotations, we'll construct the various points and lines of the proof one by one, more in the style of Euclid.

So, we have a choice to make. Either keep tranformations in the proof, or make it more traditional. We decided, tentatively, to keep the transformations.