

Math 128, Modern Geometry

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Due Today. Exercises from chapter 3: exercises 1, 2, 5b, 6, 8, 10, 15.

Read for Friday. Continue reading chapter 4.

Last time. Continued our investigation on the assumptions behind the Pythagorean theorem. Looked at wallpaper groups, transformations of the plane and symmetry groups.

Today.

Study the stereographic projection. There's a discussion in my website on *Compass Geometry* at

<http://aleph0.clarku.edu/~djoyce/java/compass/>

We'll look at the pages on "summary of inversion" and "stereographic projection."

Introduction to Klein's Erlanger Programme. In the late 19th century, Felix Klein had an idea that would help to understand the various different kinds of geometry that had been developed earlier in the 19th century. That's the century that 'noneuclidean' geometry was developed. Lobachevsky, Bolyai, and Gauss developed the first such geometry, what we now call hyperbolic geometry; Riemann developed elliptic geometry; both alternatives to Euclidean geometry. Before that Desargue came up with what was developed in the 19th century into something called projective geometry; and in the 19th century Mobius developed a geometry of the complex plane. We'll be looking at all these geometries during the course.

Klein's idea was to base a geometry, whichever one of these is under consideration, by taking as a basis the group of transformations of that geometry that preserves the properties of that geometry. For instance, if the geometry is the usual Euclidean plane geometry, then transformations of the plane that preserve distance are the appropriate transformations. That's enough, because if a transformation preserves distance, then it preserves straight lines, parallelism, angles, areas, and any other Euclidean concepts. A transformation that preserves distance is called a *rigid motion* or an *isometry*.