CLARK

Name: ____________________________

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Math 130 Linear Algebra

First Test

October 2013

You may use a calculator. Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Points for each problem are in square brackets.

1. [10] The following matrix describes a system of linear equations in five unknowns—v, w, x, y, z. What is the general solution to this system?

$$\left[\begin{array}{ccccc|c} 1 & 5 & 0 & 2 & -2 & 4 \\ 0 & 1 & 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & 1 & 7 & -2 \end{array}\right]$$

2. [10] Explain in a sentence or two why it is the case that if a subset S of a vector space V spans V, then any subset T of V that contains S (that is, $S \subseteq T$) also spans V.

3. [20] Determine if the set $S = \{(1, -1, 2), (1, -2, 1), (1, 1, 4)\}$ of three vectors in $\mathbf{R}^3$ is independent or dependent. Show your work for credit.

4. [12] Recall that a function $f : \mathbf{R} \to \mathbf{R}$ is an *odd function* if $f(-x) = -f(x)$ for each real number x. (Examples are x^3, x^5, and $\sin x$.) Determine whether or not the set of odd functions is a subspace of the set of all functions $f : \mathbf{R} \to \mathbf{R}$. After you've made your determination, write one sentence stating whether or not it is a subspace and how you made your determination.

5. On abstract vector spaces [24]. Using only the additive axioms of vector spaces prove that given two vectors $\mathbf{v}$ and $\mathbf{w}$ in the vector space

$$\mathbf{v} + \mathbf{w} = 0 \text{ if and only if } \mathbf{w} = -\mathbf{v}.$$

The four additive axioms are listed at the bottom of this page.

Work out your proof on the back of one of the test pages, then copy it here when you're satisfied you have it correct. Carefully write your proof so that you point out every use of every axiom. Fully parenthesize your expressions so that you can indicate when and where you use commutativity and associativity.

1. Vector addition is commutative: $\mathbf{v} + \mathbf{w} = \mathbf{w} + \mathbf{v}$ for all vectors $\mathbf{v}$ and $\mathbf{w}$;

2. Vector addition is associative: $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$ for all vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$;

3. There is a vector, denoted $\mathbf{0}$ and called the *zero vector*, such that $\mathbf{v} + \mathbf{0} = \mathbf{v} = \mathbf{0} + \mathbf{v}$ for each vector $\mathbf{v}$;

4. For each vector $\mathbf{v}$, there is another vector, denoted $-\mathbf{v}$ and called the *negation* of $\mathbf{v}$, such that $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$.

6. [24; 4 points each] True/false. For each sentence write the whole word "true" or the whole word "false". If it's not clear whether it should be considered true or false, you may explain in a sentence if you like.

__________ **a.** Subsets of linearly dependent sets are linearly dependent.

__________ **b.** Every system of linear equations has a solution.

__________ **c.** The intersection of any two subspaces of a vector space is also subspace of that vector space.

__________ **d.** If three nonzero vectors are linearly dependent, then one of them is a scalar multiple of one of the others.

__________ **e.** The set $S = \{(v_1, v_2, v_3) \,|\, 3v_1 + 2v_2 - v_3 = 1 \text{ and } v_1 = v_2 + 4v_3 = 0\}$ is a subspace of $\mathbf{R}^3$.

__________ **f.** The set $S = \{ax^2 + bx + c \,|\, a, b, c \in \mathbf{R}\text{and } a \neq 0\}$ of quadratic polynomials with real coefficients is a vector space.

#1.[10]	
#2.[10]	
#3.[20]	
#4.[12]	
#5.[24]	
#6.[24]	
Total	