Name: ____________________________

Mailbox number: ___________________

Math 131 Multivariate Calculus

First Test

You may refer to one sheet of notes on this test. You may leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1-\ln 10}}$ if you like. Show all your work for credit. Points for each problem are in square brackets.

Problem 1. [18; 6 points each part] On functions of several variables.

a. Give an example of a function $\mathbf{f} : \mathbf{R} \to \mathbf{R}^3$ and another example $g : \mathbf{R}^3 \to \mathbf{R}$.

b. Give an example of a vector-valued function $\mathbf{f}$ whose domain is the set

$$\{(x, y) \in \mathbf{R}^2 | x > 0 \text{ and } y > 0\}.$$

c. Explain why all the level curves for $f(x, y) = x^2 + y^2$ at positive heights c are circles.

Problem 2. [20; 10 points each part] On limits and continuity.

a. Explain why the limit, $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$, does not exist.

b. Explain why the function $f(x, y) = \sin(3x + 2y)$ is continous throughout its domain.

Problem 3. [32; 8 points each part] On derivatives.

a. Compute the gradient ∇f if $f(x, y, z) = e^{x+yz}$.

b. Find $\dfrac{\partial^2 f}{\partial x \partial y}$ for the function f given in part **a**.

c. For the function f in part **a** determine the directional derivative in the direction $\mathbf{u} = (0, \frac{3}{5}, \frac{4}{5})$.

d. Find the derivative $D\mathbf{f}$ if $\mathbf{f}(x, y) = (x^3 + 3x^2y + 3xy^2 + y^3, \sin x + \cos y, x/y)$.

Problem 4. [15] On the chain rule. Suppose that $f : \mathbf{R}^3 \to \mathbf{R}^2$ has the derivative

$$D\mathbf{f}(x, y) = \begin{bmatrix} \sin y & x\cos y & 0 \\ 2x & 2y & 2z \end{bmatrix}$$

and $\mathbf{x} : \mathbf{R}^2 \to \mathbf{R}^3$ has the derivative $D\mathbf{f}(s, t) = \begin{bmatrix} 2s & 0 \\ 2t & 2s \\ 0 & 2t \end{bmatrix}$.

a. [5] The derivative $D(\mathbf{f} \circ \mathbf{x})(s, t)$ is a matrix. What size is that matrix?

b. [10] Find the derivative $D(\mathbf{f} \circ \mathbf{x})(s, t)$. (You may leave your answer in terms of the variables x, y, z, s, and t.)

Problem 5. [16; 4 points each part] On paths.

a. Give an example of a path $\mathbf{x} : \mathbf{R} \to \mathbf{R}^2$ that passes through the point $(2, 4) \in \mathbf{R}^2$.

b. What is its velocity as it passes through $(2, 4)$?

c. What is its speed as it passes through $(2, 4)$?

d. What is its acceleration as it passes through $(2, 4)$?

#1.[18]	
#2.[20]	
#3.[32]	
#4.[15]	
#5.[16]	
Total	