Name: ____________________________

Mailbox number:___________________

Math 131 Multivariate Calculus

First Test

Feb 2014

You may refer to one sheet of notes on this test. You may leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Points for each problem are in square brackets.

1. [18; 6 points each part] On functions.

a. Find the domain of the function $g(x,y) = \dfrac{1}{\sqrt{4-x^2-y^2}}$.

b. Find the range of the function g of part **a**.

c. Consider the function $z = f(x,y) = y - x^2$. Draw either the level curve or the contour curve for f at height $c = 2$. (Draw one or the other. It's your choice.)

2. [18; 9 points each part] Evaluate the limit, or explain why it fails to exist.

a. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2}{x^2+y^2}$

b. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3+xy^2+2x^2+2y^2}{x^2+y^2}$

3. [18; 9 points each part] On derivatives.

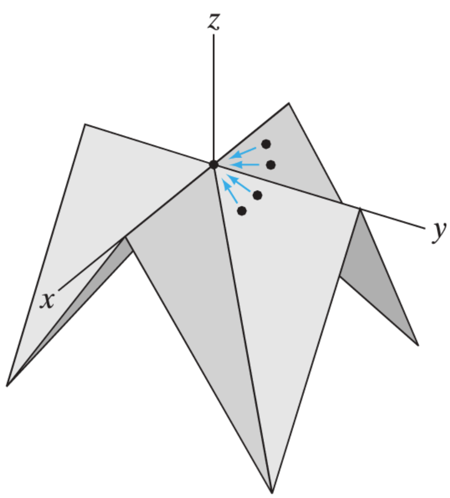
a. Let $f(x,y) = e^{x^2+y^2}$. Compute the partial derivative $\dfrac{\partial f}{\partial x}$.

b. Give an example of a function $g : \mathbf{R}^2 \to \mathbf{R}$ whose gradient is $\nabla g = (2xy, x^2)$.

4. [15] On differentiability. Consider the function whose graph $z = f(x, y)$ is displayed below. It is defined in terms of absolute values by

$$f(x, y) = |(|x| - |y|)| - |x| - |y|.$$

As you can see from the graph, it's partial derivatives evaluated at the origin are both 0, that is, $f_x(0, 0) = f_y(0, 0) = 0$. Explain why this function is not differentiable at the origin even though it's partial derivatives are both 0.



5. [15] On the chain rule. On the chain rule. Suppose that $f : \mathbf{R}^2 \to \mathbf{R}^3$ has the derivative

$$D\mathbf{f}(x, y) = \begin{bmatrix} 2xy & x^2 \\ 1 & 2y \\ y\cos xy & x\cos xy \end{bmatrix}$$

and $\mathbf{x} : \mathbf{R} \to \mathbf{R}^2$ has the derivative $\mathbf{x}' = (3t^2, t)$.

The composition $\mathbf{y} = \mathbf{f} \circ \mathbf{x}$ is a function $\mathbf{R} \to \mathbf{R}^3$. Find its derivative $\mathbf{y}'$. (Leave your answer in terms of the variables x, y, and t.)

Problem 6. [18; 9 points each part] On derivatives and directions. Let $f(x,y,z) = x(y^2+z)$ Note that the gradient of f is $\nabla f(x,y,z) = (y^2+z, 2xy, x)$

a. In which direction does f increase fastest at the point $\mathbf{a} = (1,2,3)$. (For your answer, it's enough to give a vector in that direction. It doesn't have to be a unit vector.)

b. At the point $\mathbf{a} = (1,2,3)$, evaluate the directional derivative in the direction $\mathbf{u} = (\frac{4}{9}, \frac{7}{9}, \frac{4}{9})$.

#1.[18]	
#2.[18]	
#3.[18]	
#4.[15]	
#5.[15]	
#6.[18]	
Total	