Math 131 Multivariate Calculus
Second Test
April 2014

You may refer to one sheet of notes on this test. You may leave your answers as expressions such as $e^{2\sqrt{\frac{\sin^2(\pi/6)}{1+\ln 10}}}$ if you like. Show all your work for credit. Points for each problem are in square brackets.

Problem 1. [30; 6 points each part] Calculate the velocity, speed, acceleration, and unit tangent vector of the path $\mathbf{x}(t) = (\cos t, \sin t, e^t)$.

a. Velocity.

b. Speed.

c. Acceleration.

d. Unit tangent vector.

e. Set up an integral that gives the length of that path for $1 \leq t \leq 5$. Do not evaluate the integral.
Problem 2. [15] The vector field \( \mathbf{F}(x, y, z) = (y, x, -3) \) is the gradient field \( \nabla f \) of some potential field \( f \). Find a potential function \( f : \mathbb{R}^3 \to \mathbb{R} \) for \( \mathbf{F} \).

Problem 3. [15] Set up a double integral to compute the volume of a solid whose base is the plane region \( D \) in the \((x, y)\)-plane bounded by the \( x \)-axis and the parabola \( y = 4 - x^2 \); and whose height at a point \((x, y)\) in that region is given by \( f(x, y) = \sin(x^2 + y^2) \). Be sure to sketch the region \( D \). Do not evaluate the integral.
Problem 4. [20] Consider the function \( f(x, y) = e^{-y}(x^2 - y^2) \). Its first and second partial derivatives are
\[
\begin{align*}
  f_x &= 2xe^{-y} \quad f_y = -e^{-y}(x^2 + 2y - y^2) \\
  f_{xx} &= 2e^{-y} \quad f_{xy} = -2xe^{-y} \quad f_{yy} = e^{-y}(x^2 + 4y - y^2 - 2)
\end{align*}
\]

a. Determine the two critical points of \( f \).

b. Identify the nature (max, min, saddle) of each critical point.

Parabolic coordinates. The relevant equations to convert between rectangular coordinates $(x, y)$ and parabolic coordinates $(u, v)$ are

\[
\begin{align*}
  x &= uv \\
  y &= \frac{1}{2}(u^2 - v^2)
\end{align*}
\]

A double integral can be converted from rectangular coordinates to parabolic coordinates using a Jacobian. The area differential $dA = dx\, dy$ is equal to \( \left| \frac{\partial(x, y)}{\partial(u, v)} \right| \, du\, dv \).

Determine the Jacobian

\[
\left| \frac{\partial(x, y)}{\partial(u, v)} \right| = \]

\[
\begin{array}{|c|c|c|}
\hline
#1 & [30] \\
#2 & [15] \\
#3 & [15] \\
#4 & [20] \\
#5 & [20] \\
\hline
\text{Total} & & \\
\hline
\end{array}
\]