CLARK

Name: ____________________________

Mailbox number:____________________

Math 131 Multivariate Calculus

Second Test

April 2014

You may refer to one sheet of notes on this test. You may leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Points for each problem are in square brackets.

Problem 1. [30; 6 points each part] Calculate the velocity, speed, acceleration, and unit tangent vector of the path $\mathbf{x}(t) = (\cos t, \sin t, e^t)$.

a. Velocity.

b. Speed.

c. Acceleration.

d. Unit tangent vector.

e. Set up an integral that gives the length of that path for $1 \le t \le 5$. Do not evaluate the integral.

Problem 2. [15] The vector field $\mathbf{F}(x, y, z) = (y, x, -3)$ is the gradient field ∇f of some potential field f. Find a potential function $f : \mathbf{R}^3 \to \mathbf{R}$ for $\mathbf{F}$.

Problem 3. [15] Set up a double integral to compute the volume of a solid whose base is the plane region D in the (x, y)-plane bounded by the x-axis and the parabola $y = 4 - x^2$; and whose height at a point (x, y) in that region is given by $f(x, y) = \sin(x^2 + y^2)$. Be sure to sketch the region D. Do not evaluate the integral.

Problem 4. [20] Consider the function $f(x,y) = e^{-y}(x^2 - y^2)$. Its first and second partial derivatives are

$$f_x = 2xe^{-y} \qquad f_y = -e^{-y}(x^2 + 2y - y^2)$$

$$f_{xx} = 2e^{-y} \qquad f_{xy} = -2xe^{-y} \qquad f_{yy} = e^{-y}(x^2 + 4y - y^2 - 2)$$

a. Determine the two critical points of f.

b. Identify the nature (max, min, saddle) of each critical point.

Problem 5. [20] On change of variables and the Jacobian.

Parabolic coordinates. The relevant equations to convert between rectangular coordinates (x, y) and parabolic coordinates (u, v) are

$$x = uv \qquad u = \sqrt{\sqrt{x^2 + y^2} + y}$$
$$y = \tfrac{1}{2}(u^2 - v^2) \qquad v = \sqrt{\sqrt{x^2 + y^2} - y}$$

A double integral can be converted from rectangular coordinates to parabolic coordinates using a Jacobian. The area differential $dA = dx\,dy$ is equal to $\left|\dfrac{\partial(x, y)}{\partial(u, v)}\right| du\,dv$.

Determine the Jacobian

$$\left|\frac{\partial(x, y)}{\partial(u, v)}\right| =$$

#1.[30]	
#2.[15]	
#3.[15]	
#4.[20]	
#5.[20]	
Total	