CLARK UNIVERSITY 1887

Math 217/Econ 360

First Test Extra Credit

Oct 2014

3 points each problem.

8. In order to win an election, a political candidate must win districts I, II, and III. Polls have shown that

the probability of winning both I and III is 0.55,
the probability of losing II but not I is 0.34, and
the probability of losing II and III but not I is 0.15.

Use these three probabilities to determine the probability that the candidate will win all three districts.

Since $P(I^c \cap II^c \cap III^c) = 0.15$, But $P(I \cap II^c) = 0.34$, therefore, $P(I \cap II^c \cap III) = 0.34 - 0.15 = 0.19$.

Since $P(I \cap III) = 0.55$, and $P(I \cap II^c \cap III) = 0.19$, therefore, $P(I \cap II \cap III) = 0.55 - 0.19 = 0.36$.

9. A fair die is tossed nine times. What is the probability of observing exactly two 3's, exactly three 1's, and exactly four 5's (and therefore no 2's, 4's, or 6's)?

How many ways are there of getting a sequence of length 9 containing exactly two 3's, exactly three 1's, and exactly four 5's? First choose 2 of the 9 tosses to be 3's, then choose 3 of the remaining 7 tosses to be 1's, then let the remaining 4 tosses be 2's. That gives $\binom{9}{2}\binom{7}{3}$ ways. Alternatively, it's the multinomial coefficient $\binom{9}{2,3,4}$. Both equal $\dfrac{9!}{2!3!4!}$.

There are 6^9 possible sequences in all. Therefore the probability is $\dfrac{9!}{2!3!4!}\,\dfrac{1}{6^9}$. Numerically, that turns out to be 0.00128.

Note that the three observations are not independent. If two 3's are observed, that changes the probabilities of observing three 1's. That means that you can't just compute the probabilities of the three observations and multiply them together.

10. An urn contains 8 green balls, 10 yellow balls, and 12 red balls. Six balls are removed from the urn (without replacement). Given that no green balls are chosen, determine the conditional probability that there are exactly 2 yellow balls among the chosen 6.

The easiest way to compute the conditional probability is to reduce the sample space by ignoring the green balls since it's given that no green balls are chosen.

There are 10 yellow and 12 red ones. Of six balls chosen at random, what's the probability there are exactly 2 yellow ones?

You could use a hypergeometric distribution or compute it directly. Here are the direct computations. The number of ways of choosing 6 balls out of 22 is $\binom{22}{6}$, while the number of ways of choosing 2 out of 10 yellow balls is $\binom{10}{2}$ and the number of ways of choosing 4 out of 12 red ones in $\binom{12}{4}$. Therefore, the answer is

$$\frac{\binom{10}{2}\binom{12}{4}}{\binom{22}{6}} = \frac{45 \cdot 495}{74613} \approx 0.2985$$