Seventh assignment
Math 217 Probability and Statistics
Prof. D. Joyce, Fall 2014

1. Choose a number B at random from the interval $[0,1]$ with uniform density. Find the probability that

a. $\frac{1}{3} < B < \frac{2}{3}$

b. $|B - \frac{1}{2}| \leq \frac{1}{4}$.

c. $B < \frac{1}{4}$ or $1 - B < \frac{1}{4}$.

d. $3B^2 < B$.

2. Let x and y be chosen uniformly and independently at random from the interval $[0,1]$. Which pairs of the following events are independent?

a. $x > 1/3$.

b. $y > 2/3$.

c. $x > y$.

d. $x + y < 1$.

3. Let X be a random variable with cumulative distribution function F. The *median* of X is the value m for which $F(m) = \frac{1}{2}$. Then $X < m$ with probability $\frac{1}{2}$ and $X > m$ with probability $\frac{1}{2}$. Find m if X is

a. uniformly distributed over the interval $[a,b]$.

b. exponentially distributed with parameter λ.

4. Let X be a random variable normally distributed with parameters $\mu = 70$, $\sigma = 10$. Estimate

a. $P(X > 50)$.

b. $P(X < 60)$.

c. $P(X > 90)$.

d. $P(60 < X < 80)$.

5. A final examination at Podunk University is constructed so that the test scores are approximately normally distributed, with parameters μ and σ. The instructor assigns letter grades to the test scores as shown below (this is the process of "grading on the curve").

Test Score	Letter grade
$\mu + \sigma < x$	A
$\mu < x < \mu + \sigma$	B
$\mu - \sigma < x < \mu$	C
$\mu - 2\sigma < x < \mu - \sigma$	D
$x < \mu - 2\sigma$	F

What fraction of the class gets A, B, C, D, F?

6. Let X_1 and X_2 be independent random variables with common distribution

$$p_X = \begin{pmatrix} 0 & 1 & 2 \\ 1/8 & 3/8 & 1/2 \end{pmatrix}.$$

Find the distribution of the sum $X_1 + X_2$.

7. Let X and Y be independent random variables defined on the space Ω, with density functions f_X and f_Y, respectively. Suppose that $Z = X + Y$. Find the density f_Z of Z if

$$f_X(x) = f_Y(x) = \begin{cases} 1/2, & \text{if } -1 \leq x \leq +1, \\ 0, & \text{otherwise.} \end{cases}$$

8. Suppose again that $Z = X + Y$. Find f_Z if

$$f_X(x) = f_Y(x) = \begin{cases} x/2, & \text{if } 0 < x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

Math 217 Home Page at
`http://math.clarku.edu/~djoyce/ma217/`