CLARK UNIVERSITY

Math 218 Mathematical Statistics

Second Test

April 2016

This is an open-book, open-note test, and you may use a calculator. You may leave your answers as expressions such as $\binom{8}{4}\frac{e^{1/3}}{\sqrt{2\pi}}$ if you like.

You may do the problems in any order that you like, but please start your answers to each problem on a separate page of the bluebook. Please write or print clearly. I suggest you answer question 0 last.

0. This is a partial take-home test. Choose one of the following four problems to take home and do by next class. Do not discuss the text with anyone, but you may ask me questions. You may consult our text book and and class notes, but don't consult other books or the internet.

Which of the following four questions are you choosing as a take-home question?

1. [25] A British study (*Sleep*, Jan 1989) measured systematically the effect of sleep loss on creative faculties and the ability to deal with unfamiliar situations. In the study, 12 healthy college students, deprived of one night's sleep, received an array of tests intended to measure thinking time, fluency, flexibility, and originality of thought. The overall test scores of the sleep-deprived students were compared to the average score one would expect from students who received their accustomed sleep. Suppose that the overall scores of the 12 sleep-deprived students had a mean of $\overline{x} = 63$ and a standard deviation of 17. (Lower scores are associated with a decreased ability to think creatively.)

a. [5] Consider the hypothesis H_0 that the true mean score of sleep-deprived subjects is 80, the same mean score of subjects who received sleep prior to taking the test, as opposed to the alternative hypothesis H_1 that sleep-deprived subjects have depressed scores. Use $\alpha = .05$. First, what kind of test is this? Circle one.

(i). upper one-sided

(ii). lower one-sided

(iii). two-sided

b. [5] Second, what kind of statistic will you use? Circle one.

(i). Z-statistic (i.e., normal)

(ii). T-statistic

(iii). χ^2

(iv). F-statistic

c. [10] Carry out the test to see if the data are enough to reject H_0. (Show your work.)

d. [5] What assumptions are required for the hypothesis test of part a to be valid?

2. [25] Refer to exercise 6 in chapter 7, page 263. Coffee cans are to filled with 16 oz. of coffee. The mean content of cans filled on a production line is monitored. It is known from past experience that the standard deviation of the contents is 0.1 oz. A sample of 9 cans is taken every hour and their mean content is measured.

a. [5] Set up the hypotheses to test whether the mean content is 16 oz. Should the alternative be one-sided or two-sided? Why?

b. [5] Give a decision rule in terms of the sample mean $\overline{x}$ for a 0.05-level test. Describe your rule in the form: Reject H_0 if $\overline{x}$ does not lie in some interval.

c. [7] If the true mean content during a particular period is 16.1 oz., what is the probability that the test derived in part b will correctly detect this deviation from the target value of 16 oz.?

d. [8] How many cans should be sampled to assure 90% power in part c?

3. [25] Tax assessors. In response to a complaint that a particular tax assessor (X) was biased, an experiment was conducted to compare the assessor named in the complaint with another assessor (Y) in the same office. Eight properties were selected, and each was assessed by both assessors. The assessments (in thousands of dollars) are shown in the table

Property	Assessor X	Assessor Y
1	76.3	75.1
2	88.4	86.8
3	80.2	77.3
4	94.7	90.6
5	68.7	69.1
6	82.8	81.0
7	76.1	75.3
8	79.0	79.1

With this data, the mean assessment for X is $\overline{x} = 80.775$, the mean assessment for Y is $\overline{y} = 79.287$, so the mean difference is $\overline{d} = \overline{x} - \overline{y} = 1.487$. Also the sample standard deviation for X is $s_X = 7.994$, that for Y is $s_Y = 6.851$, and the sample standard deviation for the difference is $s_d = 1.491$.

a. [5] What are the assumptions for this experiment to have a matched pairs design?

Assume in parts b and c that those assumptions are met.

b. [10] Determine a 2-sided confidence interval for the mean of the population difference μ_D which is $\mu_X - \mu_Y$ at the 95% confidence level. (Show your work and use standard notations.)

c. [10] Does the data allow you to reject the null hypotheses that the two assessors tend to give the same average assessments? Explain in a short sentence why.

4. [25] On a family of conjugate priors for Bayesian statistics.

a. [10] Let $\alpha > 1$ and $\beta > 0$. Show that the function

$$f_{\alpha\beta}(x) = \begin{cases} \frac{(\alpha-1)\beta^{\alpha-1}}{x^\alpha} & \text{if } x \geq \beta \\ 0 & \text{if } x < \beta \end{cases}$$

is a density function.

b. [15] Consider the family of uniform distributions on $[0, \theta)$ parameterized by θ. The density function for such a distribution is

$$f(x) = \begin{cases} 1/\theta & \text{if } x \in [0, \theta] \\ 0 & \text{if } x > \theta \end{cases}$$

Suppose that the prior distribution on the parameter θ has density $f_{\alpha\beta}$ described in part **a**. As described in our notes on Bayesian statistics, the posterior distribution on θ is

$$f(\theta|\mathbf{x}) \propto f(\mathbf{x}|\theta) f_{\alpha\beta}(\theta).$$

As mentioned in example 15.4, page 617, of our text

$$f(\mathbf{x}|\theta) = \begin{cases} 1/\theta^n & \text{if } x_{\max} \in [0, \theta] \\ 0 & \text{otherwise} \end{cases}$$

where $x_{\max}$ is the maximum of the values $x_1, \ldots, x_n$ in the sample. Show that the posterior distribution $f(\theta|\mathbf{x})$ is another distribution of the form found in part **a**, and determine what the new α and new β for this posterior distribution.