Three books are reviewed in this issue:


2. **Handbook of Computational Social Choice**, edited by Felix Brandt, Vincent Conitzer, Ulle Endriss, Jérôme Lang, and Ariel D. Procaccia. A collection of detailed articles on the emerging area of computational aspects of collective decision making. (I can’t help but note that fellow SIGACT columnist Lane Hemaspaandra is a contributor to this volume.) Reviewed by S.V. Nagaraj.

3. **Ramsey Theory for Discrete Structures**, by Hans Jürgen Prömel. A book which, if you know some Ramsey Theory, will likely teach you more of it, including some recent developments. At any rate, it taught the reviewer things in Ramsey Theory that even he didn’t know. Review by Bill Gasarch.

As always, the books listed on the subsequent page are available for review. Please send me an email if you’re interested in any of them. I’m also always happy to take suggestions not on the list!
BOOKS THAT NEED REVIEWERS FOR THE SIGACT NEWS COLUMN

Algorithms

1. Tractability: Practical approach to Hard Problems, Edited by Bordeaux, Hamadi, Kohli
2. Recent progress in the Boolean Domain, Edited by Bernd Steinbach
3. Compact Data Structures, by Gonzalo Navarro
4. Algorithms and Models for Network Data and Link Analysis, by François Fouss, Marco Saerens, and Masashi Shimbo

Programming Languages

1. Practical Foundations for Programming Languages, by Robert Harper

Miscellaneous Computer Science

1. Elements of Parallel Computing, by Eric Aubanel
2. CoCo: The colorful history of Tandy’s Underdog Computer by Boisy Pitre and Bill Loguidice
3. Introduction to Reversible Computing, by Kalyan S. Perumalla
4. A Short Course in Computational Geometry and Topology, by Herbert Edelsbrunner
5. Network Science, by Albert-László Barabási
6. Actual Causality, by Joseph Y. Halpern
7. Partially Observed Markov Decision Processes, by Vikram Krishnamurthy
8. The Power of Networks, by Christopher G. Brinton and Mung Chiang

Computability, Complexity, Logic

1. The Foundations of Computability Theory, by Borut Robič
2. Models of Computation, by Roberto Bruni and Ugo Montanari

Cryptography and Security

1. Cryptography in Constant Parallel Time, by Benny Applebaum
3. A Cryptography Primer: Secrets and Promises, by Philip N. Klein
Combinatorics and Graph Theory

1. *Finite Geometry and Combinatorial Applications*, by Simeon Ball
2. *Introduction to Random Graphs*, by Alan Frieze and Michał Karoński
4. *Combinatorics, Words and Symbolic Dynamics*, Edited by Valérie Berthé and Michel Rigo
5. *Words and Graphs*, by Sergey Kitaev and Vadim Lozin

Miscellaneous Mathematics and History

1. *Professor Stewart’s Casebook of Mathematical Mysteries* by Ian Stewart
Communication Networks is a textbook for a graduate course on the theoretical issues in Computer Networks and Internet that have been approached with methods from non-linear optimization, control theory, and queuing theory. I was keen to review this book since I have taught an advanced course on Computer Networks that combines algorithmic and experimental topics. Although my course follows a different perspective than the book’s, I have often had trouble to find material that is accessible and suitable for student reading and that covers some of the topics in this textbook.

As I was reading the textbook, the main question that was hovering in my mind was whether the book would be useful beyond its original “optimization, control, and stochastic networks perspective”:

Is “Communication Networks” suitable for an advanced network course with an algorithmic and experimental perspective?

As I summarize the book’s contents, I will point out parts of the book that are especially useful, and then I will round up my conclusions.

1 Congestion Control

A central topic in this textbook (and in the Internet) is congestion control, so I will start with its definition and a quick overview of its formulation as a convex optimization problem. Let $G = (V, E)$ be a directed graph representing the Internet topology: vertices are end-points or intermediate routers, and edges are communication links between vertices. An edge $e$ is associated with a positive capacity $c_e$ that represents the bandwidth on the communication link $e$. Let $\mathcal{P}$ be a set of directed paths in $G$, each of which represents the communication links used by an end-to-end communication flow. For example, a path $p \in \mathcal{P}$ could represent the set of links from a server to a client. Congestion control is defined as the convex optimization problem:

\[
\max_{x_p} \sum_{p \in \mathcal{P}} U_p(x_p) \\
\text{s.t.} \sum_{p : e \in p} x_p \leq c_e \quad (e \in E) \\
x_p \geq 0 \quad (p \in \mathcal{P})
\]
where \( x_p \) is the *sending rate* along the path \( p \in \mathcal{P} \), and \( U_p \) is the strictly concave, increasing, and continuously differentiable *utility function* of the flow along \( p \). The congestion control problem owes its name to its primary practical objective at the time it was formulated, namely the problem of avoiding congested links, as per constraint (2). In congestion control, the optimal (or near-optimal) sending rates \( x_p^* \) are converted into a schedule of packet transmissions. For example, during time slot \( t \), \( a_p(t) \) packets are sent along \( p \), where \( a_p(t) \) is a (Poisson) random variable with mean \( x_p^* \).

Although there are many techniques for solving convex optimization problems, congestion control is especially challenging due to the requirement that it must be solved in a fully distributed environment such as the Internet. In particular,

- In the Internet, end-points and routers (corresponding to the vertices) have different roles and only a local view of the global state. For example, routers should not keep track of the individual paths crossing a link (no *per-flow state*).
- Congestion control operates as fully asynchronous algorithm without a global notion of a wall-clock time.
- In addition to the data transmission, routers and end-points can also communicate housekeeping information (the so-called *control plane*). However, this additional communication is typically kept to a minimum.
- To simplify end-point error recovery, packets should be delivered at the destination in the same order in which they were originally transmitted at the source (infrequent *out-of-order delivery*).
- The path set \( \mathcal{P} \) changes dynamically.

## 2 Summary of Contents

**Chapter 1:** The “Introduction” is a quick overview to key concepts in computer networks, such as the organization of Internet software as a stack of layers. The introduction is a quick review for students who have already taken an undergraduate or graduate course in networking, but it is not meant to be a replacement for a networking course. It can also be used for students from theoretical disciplines, such as a mathematics or systems engineering, to gain a quick introduction to the aspects of networking that are relevant to the topics covered in the book.

**2.1 Part I**

The chapters in Part I discuss important facets of Internet protocols, performance, and algorithms. Part I starts with a top-down approach: it discusses global congestion control first (transport layer), then statistical multiplexing at links (network and data link layer), and finally packet scheduling within a router switch fabric (hardware) and in wireless networks (MAC sublayer). It then revisits congestion control with an eye toward cross-layer optimization by integrating congestion control with routing and packet forwarding. A chapter on peer-to-peer networks concludes Part I.

**Chapter 2:** “Mathematics of Internet Architecture” is an introduction to convex optimization and its application to the problem of congestion control. In spite of the title, the reader will not find a comprehensive view of the many theoretical facets of Internet algorithms. For example, there is no discussion of, say,
cryptography or data structures. Rather, the focus is primarily on congestion control. This chapter starts with background information on convex optimization (e.g., Karush-Kuhn-Tucker conditions) and dynamic system stability (Lyapunov theorem), and applies these notions to congestion control. The chapter has a nice discussion of fairness, of the Vickrey-Clarke-Groves mechanism to ensure that flows declare their true utility, and of the Nash equilibrium for Kelly’s mechanism. Although most of the discussion assumes no network latency, communication delays are demonstrated in a simple example.

Typical Computer Science students will have only an approximate understanding of optimization, and their background would probably be more slanted toward combinatorial optimization. Therefore, it is probably a good idea to allocate enough lecture time to cover this chapter, since convex optimization may be relevant not only for congestion control but also to other problems that students may encounter in their studies. Similarly, the discussions of utility functions, fairness, and mechanisms are useful not only in the context of congestion control but also in (online) algorithms, and it is worthwhile to spend lecture time to develop a solid understanding of these concepts.

Chapter 3: “Links: statistical multiplexing and queues” is an introduction to discrete-time Markov chains and queuing theory. The chapter starts with an introduction to the Chernoff bound, which is obviously important for students to know not only in the context of communication networks, but also in other applications, such as randomized algorithms. The chapter also provides an introduction to packet buffering, statistical multiplexing, and weighted fair queuing, which are topics typically covered in undergraduate courses in networking, but may be a welcome addition for students with a more theoretical background. Some examples assume that packet arrivals are i.i.d. Bernoulli processes. I would skip these examples in an algorithmic course, because they have a stochastic bent and the assumption is questionable in practice in the Internet.

Chapter 4: “Scheduling in packet switches” considers the problem of matching input and output queues in a router or switch so as to maximize the packet arrival rate that can be processed by the switch fabric. The analysis again assumes stochastic arrivals. In general, the probabilistic solution to switch scheduling requires cubic time, and lower run-time algorithms are discussed.

Chapter 5: “Scheduling in wireless networks” discusses the issues arising from scheduling packet transmission in wireless networks. The main issues are medium contention among sources, potentially poor reliability of the communication channel, and the requirement that the scheduling algorithm should be distributed. Consequently, wireless access control can be significantly more complex than the problem of scheduling the switching fabric, which was examined in the previous chapter. The chapter starts with an overly simple scenario (scheduling on downlinks only) and builds up toward the more complex and realistic scenario of ad-hoc WiFi networks. The algorithms in the previous chapter and their analyses are extended to the wireless case.

Chapter 6: “Back to network utility maximization” combines the results of congestion control (Chapter 2) and ideas from scheduling in wireless networks. The insight is to solve the congestion control optimization problem assuming that the capacities $c_e$ are a function of routing and of transmission scheduling. In this new formulation, the link capacity constraints split into two new constraint sets: the first set expresses packet counts in terms of bandwidth and reliability, and the second states that the number of packets arriving or generated at a node must not exceed the number of packets departing that node. The second constraint set can be relaxed via Lagrangian multipliers $\phi$. If $\phi$ is a constant, congestion control splits into multiple
optimization problems which are independent of each other and, importantly, such that each subproblem can be solved at a network node (source or router) using only information that is locally available. In particular, each intermediate node solves an optimization problem that can be interpreted as the problem of finding an optimal routing and forwarding schedule. In fact, the Lagrangian multipliers \( \phi \) are not constants, and are the vehicle by which network nodes exchange information in the process of solving the combined optimization problem. The resulting algorithm is theoretically intriguing, but the reader should keep in mind that its properties are only proven under probabilistic assumptions on packet transmissions \( a_p(t) \), that the forwarding algorithm maintains per-flow queues, and that dynamic routing leads to out-of-order delivery.

Chapter 7: “Network protocols” reviews some of the congestion control and routing algorithms in actual Internet use and compares them with the theoretical results of the previous chapters. As the chapter introduction points out, the historical evolution was in fact the exact opposite. For example, congestion control was introduced to avoid congestion collapse in the early Internet through implementation and experimentation, and found a theoretical framework only a few years later. Even though the theoretical formulation was a late arrival, it was important because it spawned renewed engineering and new algorithms, such as TCP FAST, which is covered in this chapter. In short, the chapter shows that the congestion control algorithms TCP Reno can be viewed as an approximation of the theoretical congestion control problem in which \( U_p(x_p) \propto \arctan x_p \) up to an additive constant, at least in some circumstances. The chapter also reviews standard algorithms for routing and for medium access control in wireless networks. In these cases, the connection between theory and practice is less clear, and in particular it is a topic of ongoing research to combine the algorithms in Chapter 5 (scheduling based on independent sets) with the CSMA/CA (RTS/CTS) protocol. This chapter is definitely something that I wished I had the first time I taught my course. The relationship between theory and practice is complicated by the fact that practice historically preceded theory, and that the link between the two is discussed in many research papers, but not in a form that is concise and suitable for a relatively short lecture and for study material. It would have also been useful in research since it is often helpful to know the main idea and proof outlines more than the scenario-specific details that are covered in a research paper.

Chapter 8: “Peer-to-peer networks” introduces and analyzes Distributed Hash Tables (DHT), structured streaming, and gossip processes. As an example, BitTorrent is also summarized. The topics are important for students to know and I was glad to see them covered in this textbook. Furthermore, some of these topics are relevant beyond communication networks, as is the case of randomized algorithms for data structures and of gossip processes, making the chapter even more relevant in a curriculum.

2.2 Part II

The second part of the book is a review of various topics that are the starting point of network research within a stochastic perspective.

Chapter 9: “Queuing theory in continuous time” reviews classical results in queuing theory, from M/M/1 to GI/GI/1 queues and their applications to the analysis of congestion stability under connection arrivals and departures, distributed admission control, and BitTorrent download time. It also applies queuing theory to the problem of congestion stability under connection arrivals and departures assuming that file sizes are exponentially distributed.
**Chapter 10:** “Asymptotic analysis of queues” deals with two extreme scenarios in queuing theory, one when the arrival rate is close to link bandwidth and the other when rare events occur. The chapter generally assumes that $a_p(t)$ are i.i.d. random variables across $p$ and $t$. The first part of the chapter covers the heavy traffic scenario, which assumes that link bandwidth exceeds arrival rates slightly, hence leading to long queues. The analysis is carried out under a technical assumption, which is shown to hold for Poisson traffic. The second part of the chapter deals with the tails of the queue length distribution.

**Chapter 11:** “Geometric random graph models of wireless networks” considers the problem of maximizing the throughput in a wireless network with an increasing number $n$ of interfering wireless endpoints. First, it shows that the throughput is $O(1/\sqrt{n})$ if nodes are placed arbitrarily in the unit square, as long as the average distance of source-destination pairs is independent of $n$. The argument relies on the Hoeffding bound, which is introduced at the beginning of the chapter. Then the book presents an algorithm that nearly achieves the lower bound as long as communication nodes are placed uniformly in the unit square.

### 3 Opinion

“Communication networks” is a great textbook for students interested in the theoretical aspects that straddle the areas of optimization, control theory, and queuing. It is also undoubtedly valuable for more applied students seeking insight and underlying principles to the complex and often confusing behavior of the Internet. I liked that the book has a very good introduction to every chapter, with a list of key questions that will be addressed in the chapter, a summary at the end, and a set of useful exercises. I liked the way proofs are typographically marked throughout the textbook.

I started my review with a question in mind, namely, if I was going to adopt this textbook for an advanced Networks course that has an algorithmic rather than control theory and stochastic perspective. In short, I would certainly cover in depth certain chapters, while skipping other ones. I should remark that these comments are not meant to detract from the importance of the topics (they are) or clarity of the book (it is), but only to address the somewhat parochial question of whether the textbook would be useful to me and my students for teaching, studying, and research. In particular, I would talk in depth about the theory of congestion control (chapter 2) and its relation to practice (chapter 7). It would also be interesting to cover the cross-layer interaction of congestion control, routing, and packet forwarding (chapter 6). Finally, I would also cover DHT, structured streaming, and gossip (chapter 8). These topics are covered in a way that is clear and concise, and extremely useful both as a course textbook and as an introductory research reading. Having struggled for years to find appropriate references on this topic, I am glad to have found this resource.

Since the book is focused primarily on theory, if a course also has a more practical component, it would obviously have to include additional topics, such as network emulation, Internet measurement, and the engineering of the transport layer. Similarly, an algorithmic course should use the relevant chapters in this book, but would benefit from the coverage of other topics or of the same topics but from a different perspective. For example, the topics in chapter 10 could be covered in an alternative form in which the problem is formulated as load balancing of permanent jobs on two machines. As another example, Bloom filters could be added to the coverage of chapter 4.

In my opinion, important omissions are the topics of self-similarity and long-range dependence. These issues would have fit within the book’s stochastic perspective, have been validated in Internet measurement, and are critical for addressing network issues such as congestion control and bandwidth provisioning.
4 Conclusions

“Communication Networks” is a remarkable textbook, and a clear and concise reference for research in topics ranging from congestion control to distributed hash tables. Many of its chapters are extremely invaluable also to those who have a research perspective other than one in optimization, control, and stochastic processes.
1 Introduction

Social choice theory is an area of economics that studies collective decision making. It provides mathematical models and guidelines for making the right choices. Examples of collective decision making include sharing a cake among friends and voting in an election. Computational social choice is a discipline which may be considered to be at the crossroads of economics and computer science. This handbook on computational social choice deals with the computational aspects of collective decision making. It contains contributions from thirty-six eminent researchers in the field of computational social choice. The key areas covered by the book are voting, fair allocation of divisible as well as indivisible goods, coalition formation, and miscellaneous topics such as judgement aggregation and knockout tournaments. The book is available in hardcover and Adobe eBook Reader formats. The ISBNs are 9781107060432 (hardcover) and 9781316490631 (ebook). They are priced at US $59.99 and US $48, respectively.

2 Summary

The book contains four parts comprising 19 chapters, references to the literature, and an index. The first chapter offers a gentle introduction to computational social choice. It provides a short glimpse into the history of social choice theory and a brief outline of the book. The chapter also lists topics that are not covered by the book. It briefly introduces concepts from theoretical computer science such as computational complexity, and linear and integer programming. This chapter is authored by the five editors of the book, i.e., Brandt, Conitzer, Endriss, Lang and Procaccia.

The first part of the book focusses on voting. There are nine chapters in this part which constitutes almost half of the book. This part reflects the substantial amount of research that has gone into voting theory. The first chapter offers an introduction to the theory of voting. Several important voting rules such as those of Borda, Copeland, and Kemeny are described. The axiomatic approach is introduced. In the axiomatic approach, the basic properties of a system are phrased as axioms. Several characterization results and impossibility theorems are studied. Emphasis is also placed on strategic manipulation in elections. Marquis de Condorcet was a French nobleman who developed an important principle according to which any alternative that beats all other alternatives in direct pairwise contests should be considered the winner of the election.
Fishburn classified voting rules in order to give a structure to Condorcet extensions. These extensions follow the Condorcet principle. The Condorcet extensions were grouped by Fishburn into three classes: C1, C2, and C3. The chapter includes definitions, several theorems, corollaries, propositions, lemmas, and proofs for some of these extensions.

The second chapter is on tournament solutions and is concerned with voting rules that only depend on pairwise majority comparisons. Such rules are called C1 functions. Directed graphs may be used for representing pairwise comparisons. These graphs become tournaments when there is an odd number of voters with linear preferences. The chapter looks at important results, and various tournament solutions and their extensions to weak tournaments. Efficient computation of tournament solutions is also emphasized. The third chapter focuses on weighted tournament solutions. Here the focus is on voting rules that are only contingent on weighted pairwise majority comparisons. Such rules are known as C2 functions. Several important voting rules fall in this category. They include Kemeny’s rule, the maximin rule, the ranked pairs method, Schulze’s method, and Borda’s rule. Algorithms, parameterized algorithms, and complexity theoretic results are also reported. The focus of the fourth chapter is on Dodgson’s rule and Young’s rule. Dodgson was a mathematician best known by his pen name Lewis Carroll. Dodgson’s rule and Young’s rule belong to the class C3 with computationally hard winner determination problems. Algorithmic methods to circumvent this intractability are therefore studied. They include approximation algorithms, fixed-parameter tractable algorithms, and heuristics.

The fifth chapter is on barriers to manipulation in voting. A voter may potentially misreport his/her preferences in order to get better results for himself/herself. The impossibility result of Gibbard-Satterthwaite says that manipulation cannot be completely avoided in general. In order to circumvent this, the authors discuss ways of erecting computational barriers to manipulation. The sixth chapter is on control and bribery in voting. Control and bribery are variants of manipulation that may perhaps be carried out by the election organizer. In control, voters may be added or deleted. Bribery alters the voter’s preferences without altering the structure of the election. The chapter talks about results related to the computational complexity of the control and bribery problems under various voting rules. It is stated that although control and bribery are often used to attack elections, they have some positive applications which deserve to be explored. The seventh chapter is on rationalization of voting rules. The well known approach in social choice justifies a particular voting rule as the axiomatic one. Suitable properties of a voting rule are named and then a rule that has all of these properties is selected. However, there are other approaches that have been studied. One such approach is the maximum likelihood approach. In this approach there is an unobserved correct outcome for which a voting rule ought to be picked out to approximate this outcome. Another approach is the distance rationality approach. The chapter lists several examples, definitions, theorems, corollaries, and propositions, and a few proofs.

The eighth chapter is on voting in combinatorial domains. Here the focus is on voting in domains that are the Cartesian product of several finite domains each of which represents an issue or variable or attribute. The examples for this type of voting include multiple referenda, multi-winner elections, group configuration, and group planning. The chapter begins with the need for studying such voting and describes some classes of problems. It also mentions several classes of solutions for such elections. Many illustrative examples are also provided. The ninth chapter is on incomplete information and communication in voting. This chapter looks at models of partial preferences, solution concepts, the complexity of communication and queries, voting with an uncertain set of alternatives, and social choice from a practicable position. The complexity of determining whether a particular alternative is still a possible winner after some of the voter preferences have been processed is also studied. Schemes for efficaciously extracting voter preferences for different voting rules are also explored.
The second part of the book deals with fair allocation. There are three chapters. The first chapter offers a short introduction to the theory of fair allocation. The resource allocation problem and the notion of fairness as studied in economics are discussed. The chapter includes a wide range of fairness criteria applicable to such problems. This chapter focuses on classical concepts of economics while the remaining two chapters study problems which are computational in character. The second chapter analyzes the fair allocation of indivisible goods. The topics covered include the compact representation of preferences for fair allocation problems, criteria for fairness, the tradeoff between fairness and efficiency, the algorithmic challenges in computing socially optimal allocations, complexity results, the notions of low envy and envy-freeness, and protocols for fair allocations. An interesting computational problem that is discussed here is the Santa Claus problem. Santa Claus has \( p \) gifts to allocate to \( n \) children having modular preferences. Santa Claus allocates the gifts so as to maximize the utility of the unhappiest child. It is noted that this problem remains NP-hard even in restrictive settings.

The third chapter looks at algorithms for cutting cakes and distributing them in a fair manner. The problem involves a heterogeneous resource such as a cake with different toppings. The cake has to be divided among several people who have varying preferences over different parts of the cake. The division should be subjectively fair so that each person gets a piece that he or she thinks is a fair share. The cake cutting algorithms at first sight appear to have no practical applications, however, they typify algorithms for fair allocation of divisible goods. The cake is merely a metaphor. It should be noted that the cake cutting problem is quite different from the problem of fair allocation of indivisible goods. The chapter discusses the cake cutting problem, classical algorithms, the complexity of cake cutting, envy-free cake cutting, optimal cake cutting, and presents pointers for further reading. The chapter lists some important theorems and lemmas related to cake cutting along with proofs for some of them.

The third part of the book is on coalition formation - a topic which is part of game theory. There are three chapters on matching under preferences, hedonic games, and weighted voting games. The chapter on matching under preferences considers a few applications. It looks at the setting where each side has preferences over the other side. The traditional example for this scenario is the problem of matching men to women while the non-traditional example is that of matching junior doctors to hospitals. The famous result for this problem is the Gale-Shapley algorithm. The chapter also studies other settings such as when only one side has preferences over the other. A class of problems known as house allocation problems are described. Various theorems, algorithmic results and complexity estimates are studied, and resources for further reading are mentioned.

Matching under preferences can be considered as a special case of coalition formation where only certain types of coalitions are allowed, for example, coalitions of size two. Hedonic games can be considered as a generalization of matching under preferences. Hedonic games are more universal as any coalition structure is feasible. The main concept here is that an agent’s perception of a coalition structure only reckons on the coalition he is a member of and not how the rest of the players are classified. The chapter on hedonic games introduces the notion of hedonic games, surveys solution concepts, the relationship with cooperative game theory, preference restrictions and game representations, computational aspects of coalition stability, algorithmic and complexity theoretic results, research issues, and pointers for further reading. Hedonic games have many interesting practical applications. Examples include formation of research teams, formation of coalition governments, clusters in social networks, distributed task allocation for wireless agents, and scheduling group activities. The third chapter is on weighted voting games. They model situations where voters with variable voting weights accept or reject a proposal. A coalition of agents is said to win if and
only if the sum of the weights of the coalition equals or exceeds a specified threshold. This chapter, which includes some definitions and computational properties, analyzes voter weight versus voter power, and simple games and yes/no voting systems. The chapter contains definitions and examples, complexity theoretic results, and some important theorems and lemmas along with proofs for some of them.

The fourth part looks at additional topics. There are three chapters. The chapter on judgement aggregation reviews aggregation rules, agenda characterization, related frameworks and applications in computer science. Judgement aggregation is concerned with the aggregation of judgements pertaining to the truth or falsehood of a number of statements. The statements which may possibly be related are expressed using propositional logic. The chapter concentrates on the axiomatic foundations of judgement aggregation, specific aggregation procedures, and the complexity of judgement aggregation. An interesting application to computer science that is described here is collective decision making in systems of autonomous software agents. The chapter includes definitions and examples, complexity theoretic results, and some important propositions, theorems, lemmas, and corollaries, along with proofs for some of them.

The chapter on the axiomatic approach and the Internet provides an axiomatic characterization of ranking systems, in particular, the PageRank algorithm. It focuses on trust-based recommendation and cites mechanisms for multi-level marketing, and additional applications. The chapter applies the axiomatic approach to a number of systems on the Internet including crowdsourcing and recommender systems. Knockout tournaments are common in sports yet they provide an interesting model of decision making. They specify an agenda of pairwise competitions between alternatives, in which the alternatives are eliminated (knocked out) in an iterative manner until only a single alternative remains. In knockout tournaments no ties are allowed. The chapter on knockout tournaments starts with formal definitions and properties, and studies agenda control for general knockout tournaments and for balanced trees. It also discusses further extensions.

The handbook contains a very exhaustive list of references to the literature running to over 50 pages. There is a short index which is helpful.

3 Opinion

Computational social choice covers an immense mixture of questions of theoretical relevance and practical significance. It will enhance the collaboration and swapping of ideas between computer science and social choice theory. Since the field of computational social choice is growing rapidly, a handbook such as this at this juncture is the need of the hour. The handbook is the product of the efforts of 36 outstanding members of the computational social choice community. It provides elaborate initiations to the major areas of the field. The handbook has already become an authoritative reference work and has been cited over 100 times since its publication. It contains many interesting open questions which will serve as fodder for hungry researchers. This will be aided by the very thorough list of scores of references to the literature. The book contains important ideas drawn from lots of key research papers. The chapters are largely self-contained, and have been written so that they may be read independently. The overlap of information among the chapters is minimal. The editors have ensured that the chapters are homogeneous despite authorship by researchers with varied interests. The publisher has provided a freely downloadable copy of the book on their web site for fair use. The book is suitable for teaching courses related to computational social choice, however, for pedagogy it does not include problems and exercises. The book contains many theorems and some proofs or at least outlines of proofs. In order to fully appreciate some of the results in the book, the reader should have a deep knowledge of the subject of those results. This is especially true of results from complexity theory.
The algorithmic or computational twist to social choice theory is relatively new.

The discussion regarding the future of voting at the end of the second chapter is interesting. The author of that chapter states that it is often held that the results of Arrow, Gibbard, and Satterthwaite killed the field and subsequent work amounted to picking up just the leftovers. However, the author reasons that such views were expressed well before the birth of fields such as computational social choice or judgement aggregation, and opines that impossibility results do not kill the subject of voting. The book is a treasure trove of ideas from economics and computer science. Academicians, professionals, researchers, and students in many disciplines including economics, computer science, game theory, mathematics, philosophy, and political science will gain from this approachable and self-contained handbook.
1 Introduction

Here are examples of theorems in Ramsey Theory:

1. For all 2-colorings of the edges of $K_6$ there is a monochromatic $K_3$. That is, there are three vertices such that all of the edges between them are the same color.

2. (Ramsey’s Theorem) For all $c \in \mathbb{N} \neq 0$, for all $m$, there exists a number $n \leq 2^{2m-1}$ such that for all $c$-colorings of the edges of $K_n$ there is a monochromatic $K_m$. That is, there are $m$ vertices such that all of the edges between them are the same color. (It is known that $n \geq 2^{m/2}$.)

3. For all 2-colorings of $\{1, \ldots, 9\}$ there exists a monochromatic arithmetic sequence of length 3.

4. (van der Waerden’s Theorem, VDW) For all $c, k \in \mathbb{N} \neq 0$, there exists $W = W(k, c)$ such that for all $c$-colorings of of $\{1, \ldots, W\}$ there exists a monochromatic arithmetic sequence of length $k$.

Ramsey’s Theorem and VDW’s theorem are similar philosophically: if you color a large enough object you get a nice monochromatic sub-object. Are they related mathematically? See the summary of Part II to find out!

The first book on Ramsey Theory (excluding specialized monographs) was by Graham, Rothchild and Spencer [1]; and the second was by Landman and Robertson [2]. The book under review could be entitled A Second Course in Ramsey Theory.

The book is self contained; however, it’s rough going unless you are already somewhat familiar with the subject.

2 Summary of Contents

The book is in five Parts, each one of which is subdivided into chapters. We summarize by Parts.
Part I: Roots of Ramsey Theory

The first chapter, *Ramsey’s Theorem*, proves Ramsey’s theorem for hypergraphs and the canonical Ramsey theorem (which needs the proof of Ramsey’s Theorem for hypergraphs even if all you want to prove is Canonical Ramsey for Graphs). We state both theorems.

**Definition:** Let $c, k, m \in \mathbb{N} \setminus \{0\}$. Assume you are given a $c$-coloring of the *edges* of the complete $k$-hypergraph of $m$ vertices. A subset $H$ of the vertices is *homogenous* if every edge that consists of $k$ elements of $H$ (that is, an element of $\binom{H}{k}$) is the same color.

1. **(The Hypergraph Ramsey Theorem)** Let $c, k, m \in \mathbb{N} \setminus \{0\}$. There exists $n$ such that for all $c$-colorings of the *edges* of the complete $k$-hypergraph there is a homogenous set of size $m$.

2. **(The Canonical Ramsey Theorem for Graphs)** For all $m \in \mathbb{N} \setminus \{0\}$ there exists $n$ such that any coloring of the edges (with any number of colors, though bounded by $\binom{n}{2}$ since you are coloring the edges) there exists a set $H$ of size $m$ such that either (1) $H$ is homogenous, (2) for $x, y \in H$ the color of $(x, y)$ depends only on $\min\{x, y\}$ (called a *min-homogenous set*), (3) for $x, y \in H$ the color of $(x, y)$ depends only on $\max\{x, y\}$ (called a *max-homogenous set*, or (4) all elements of $\binom{H}{2}$ are colored differently (called a *rainbow set*). To prove this one needs the 4-hypergraph Ramsey Theorem.

The second chapter, *From Hilbert’s Cube Lemma to Rado’s Thesis* takes you through many theorems that involve coloring $\mathbb{N}$ (and in one case $\mathbb{Q}$). Of particular interest: the book gives an exact condition, which we call COND, on a matrix $A$, such that the following are equivalent:

1. Matrix $A$ satisfies condition COND.
2. For all finite colorings of $\mathbb{N} \setminus \{0\}$ there exist monochromatic numbers $x_1, \ldots, x_m$ such that $A\vec{x} = 0$.
3. For all finite colorings of $\mathbb{Z} \setminus \{0\}$ there exist monochromatic numbers $x_1, \ldots, x_m$ such that $A\vec{x} = 0$.
4. For all finite colorings of $\mathbb{Q} \setminus \{0\}$ there exist monochromatic numbers $x_1, \ldots, x_m$ such that $A\vec{x} = 0$.

Part II: A Starting Point for Ramsey Theory: Parameter Sets

This is the hardest Part in the book. We discuss what is proven.

The Hales-Jewitt (HJ) theorem is a generalization of VDW theorem. From the HJ theorem one can prove, not just VDW’s theorem, but the Gallai-Witt theorem, which is a multidimensional version of VDW’s theorem. All of this is proven.

The Graham-Rothchild (GR) Theorem is a generalization of the HJ theorem!! From the GR theorem one can prove both *The Hypergraph Ramsey Theorem* and VDW’s *theorem*. Hence, this shows Ramsey’s Theorem and VDW’s Theorem are both part of the same phenomenon and not just *philosophically similar*. The GR theorem is very powerful but very abstract. We give one more corollary of GR’s theorem:

1. For all $c, m \in \mathbb{N} \setminus \{0\}$ there exists $n$ such that for all $c$-colorings of $\{1, \ldots, n\}$ there exists a $A \subseteq \{1, \ldots, n\}$ of size $m$ such that every nonempty subset of $A$ has the same color sum.

This Part also proves Canonical versions (allow infinitely many colors) of the The HJ theorem, VDW’s theorem, and The GR theorem.
Part III: Back to the Roots: Sets

This Part feels like a breath of fresh air compared to the hard abstractions in Part II. Topics are:

1. Some Exact and Some Asymptotic Ramsey Numbers. This is an easy chapter that is usually at the beginning of a course in Ramsey Theory.

2. The Paris-Harrington result about a Ramsey Theorem which is true but cannot be proved in Peano Arithmetic.

3. Product theorems. This is essentially Ramsey Theorems for bipartite, tripartite, $k$-partite graphs. Example: for all $c, m$ there exists $n$ such that for all $c$-colorings of $K_{n,n}$ there exists a monochromatic $K_{m,m}$.

4. A Quasi Ramsey Theorem. This is about the Erdős Discrepancy conjecture (not proven at the time but proven by Terry Tao in 2016). The idea is that in every long sequence of $\{-1, 1\}$ there will be some arithmetic progression where the sum is large.

5. Partition Relations for Cardinals. Ramsey Theory on infinite cardinals. Example (actually a counterexample): There is a 2-coloring of pairs of reals such that there is no homogenous set of size the continuum.

Part IV: Graphs and Hypergraphs

Consider the following triviality: if $G$ has a clique of size 6 in it then any 2-coloring of the edges of $G$ has a monochromatic $K_3$. But what if $G$’s largest clique is of size 5? 4? 3? 2? 1? IF it’s 1 or 2 then the theorem will be false. But the following is true:

1. Let $c$ be a number of colors and $F$ be a graph. There is a graph $G$ with the same max clique size as $F$ such that, for all $c$-colorings of the edges of $G$, there is a monochromatic $F$.

Most of this Part is about theorems like that. There is also material on (1) Random Graphs, (2) Rado’s graph, a graph such that any finite or countable graph is an induced subgraph, and (3) some more rather hard theorems related to the GR theorem.

Part V: Density Ramsey Theorems

The original proof of van der Waerden’s theorem gave Ackerman-like bounds on $W(k, c)$. Erdős made the following conjecture hoping it would lead to an alternative proof of VDW’s theorem with smaller bounds: If $A \subseteq \mathbb{N}$ is of positive upper density then, for all $k$, $A$ has arithmetic sequences of length $k$.

Roth proved the $k = 3$ case. Szemerédi proved the $k = 4$ case and later the general case. Szemerédi’s proof used VDW’s theorem and hence did not lead to smaller bounds. (Shelah got a better bound though not through Erdős’s conjecture, then Gowers got a much better bound by improving known proofs of Erdős’s conjecture.) Szemerédi’s Theorem is called a density theorem. His proof was purely combinatorial though difficult. Later a density version of the HJ was proven; however, the proof was not purely combinatorial. Gowers proposed a polymath project (many people contributing) to try to find a purely combinatorial proof of the HJ density theorem. The project was a success. This book presents that proof.
3 Opinion

This is a good but tough book. I would recommend the reader already know Ramsey’s theorem on hypergraphs and van der Waerden’s theorem before beginning to read it.

However, if one knows some Ramsey Theory this book will teach you more of it. There is much in this book that is interesting and not that well known. And it’s good having it all in one place. But a warning – the subject his hard, hence the book is hard.

References
