

Name: $\qquad$

Circle your instructor's name:
Broker Hill Joyce Pendharkar

## Math 121 Calculus II <br> Second Test <br> March 2015

This is a closed-book, closed-notes test. Calculators are not allowed. Please turn off your cellphone and any other electronic equipment during the test.

Leave your answers as expressions such as $e^{2} \sqrt{\frac{\sin ^{2}(\pi / 6)}{1+\ln 10}}$ if you like. Show all your work for credit. Be sure that your proofs and computations are easy to read. Points for each problem are in square brackets.

1. [16] Evaluate the following indefinite integral.

$$
\int \frac{\sqrt{x^{2}-4}}{x} d x
$$

2. $[24 ; 12$ points each part $]$ Evaluate the following indefinite integrals.
a. $\int x^{2} \sin x d x$
b. $\int \cos ^{3} x \sin ^{2} x d x$
3. [12] Evaluate the following definite integral.

$$
\int_{1}^{e} x \ln x d x
$$

4. [12] Solve the separable differential equation $\frac{d y}{d x}=e^{-y} \cos x$. Your answer should express $y$ as a function of $x$.
5. [20] Consider the rational function $\frac{7 x+7}{x^{2}+3 x-10}$.
a. [4] Factor the denominator.
b. [8] Write the rational function as a sum of partial fractions.
c. [8] Use what you found in part b to evaluate this integral.

$$
\int \frac{7 x+7}{x^{2}+3 x-10} d x
$$

6. [16; 8 points each part] The most common form of radium, radium-226, has a half life of 1601 years.
a. Write down an formula that gives the amount $y$ of radium left after a period of $t$ years when the initial amount is $A$.
b. Use that formula to determine when only $\frac{1}{3}$ of the initial amount will remain. (Leave your answer in terms of exponents and logs.)

| $\# 1 .[16]$ |  |
| :--- | :--- |
| $\# 2 .[24]$ |  |
| $\# 3 .[12]$ |  |
| $\# 4 .[12]$ |  |
| $\# 5 .[20]$ |  |
| $\# 6 .[16]$ |  |
| Total |  |

Some useful formulas

## Trig identities

$$
\begin{aligned}
& \cos ^{2} \theta=\frac{1+\cos 2 \theta}{2} \\
& \sin ^{2} \theta=\frac{1-\cos 2 \theta}{2}
\end{aligned}
$$

## Trig subs

$$
\begin{array}{|c|c|c|}
\hline x=a \sin \theta & x=a \tan \theta & x=a \sec \theta \\
d x=a \cos \theta d \theta & d x=a \sec ^{2} \theta d \theta & d x=a \sec \theta \tan \theta d \theta \\
\sqrt{a^{2}-x^{2}}=a|\cos \theta| & \sqrt{a^{2}+x^{2}}=a|\sec \theta| & \sqrt{x^{2}-a^{2}}=a|\tan \theta| \\
\hline
\end{array}
$$



Some useful integrals Most of those on page 431 of the text you should know (especially 1-7). Here are some others

$$
\begin{gathered}
\int \sec ^{2} x d x=\tan x+C \\
\int \csc ^{2} x d x=-\cot x+C \\
\int \sec x \tan x d x=\sec x+C \\
\int \csc x \cot x d x=-\csc x+C \\
\int \tan x d x=\ln |\sec x|+C \\
\int \cot x d x=\ln |\sin x|+C \\
\int \sec x d x=\ln |\sec x+\tan x|+C \\
\int \csc x d x=-\ln |\csc x+\cot x|+C
\end{gathered}
$$

