Name: ______________________________

Mailbox number: ____________________

Math 130 Linear Algebra

Final Exam

December 2013

You may use a calculator and a sheet of notes. Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Points for each problem are in square brackets.

1. [18; 6 points each part] Consider the linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ such that $T(\mathbf{e}_1) = \mathbf{e}_2$, $T(\mathbf{e}_2) = \mathbf{e}_3$, and $T(\mathbf{e}_3) = \mathbf{e}_1$.

a. What is the 3×3 matrix A that represents T, that is, $T(\mathbf{x}) = A\mathbf{x}$?

$$A = \left[\begin{array}{c|c|c} \quad & \quad & \quad \\ \hline \quad & \quad & \quad \\ \hline \quad & \quad & \quad \end{array}\right]$$

b. The matrix A^3 represents $T \circ T \circ T$, the triple composition of T. What matrix is that?

c. Is the transformation T a reflection, and if so, across which plane, or is it a rotation, and if so, then by what angle and about what axis, or is it some other transformation of space?

2. [18; 6 points each part] Consider the matrix

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

a. Find the characteristic polynomial of A.

b. Find the eigenvalues for A.

c. Choose one of the eigenvalues that you found in part **b** and determine all the eigenvectors for it.

3. [10] Find a basis and the dimension of the vector space of all vectors of the form (a, b, c, d) in $\mathbf{R}^4$, where $a + b = c + d$.

4. [12] Prove that if A is an upper triangular matrix ,then the eigenvalues of A are the elements on the main diagonal of A.

5. [10] Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 0 & 2 & 1 \end{bmatrix}$. (Check your answer if you have time.)

6. [12] Consider the linear transformation $T : \mathbf{R}^5 \to \mathbf{R}^3$ represented by the matrix

$$\begin{bmatrix} 1 & 2 & 3 & -1 & 0 \\ 2 & 1 & 0 & 3 & 2 \\ 4 & 5 & 6 & 1 & 2 \end{bmatrix}.$$

Determine the rank and nullity of T. (Show your work)

Rank = **Nullity =**

7. [21; 3 points each] True/false. For each sentence write the whole word "true" or the whole word "false". If it's not clear whether it should be considered true or false, you may explain in a sentence if you like.

___________ **a.** Given a linear transformation $T : \mathbf{R}^n \to \mathbf{R}^n$, there is a basis of $\mathbf{R}^n$ whose basis vectors are all eigenvectors.

___________ **b.** Inner products distribute over addition, that is, $\langle \mathbf{u}+\mathbf{v}|\mathbf{w}\rangle = \langle \mathbf{u}|\mathbf{w}\rangle + \langle \mathbf{v}|\mathbf{w}\rangle$.

___________ **c.** The determinant of a square matrix is equal to the determinant of its transpose.

___________ **d.** The inequality $\|\mathbf{w}-\mathbf{v}\| \leq \|\mathbf{w}\| + \|\mathbf{v}\|$ is known as Cramer's inequality.

___________ **e.** Matrix multiplication is commutative: $AB = BA$.

___________ **f.** The following set S is a basis for $\mathbf{R}^6$.

$$S = \{(3,2,0,8,-5,2), (4,3,-2,0,4,1), (-3,2,1,4,5,2), (2,3,-2,1,0,0), (0,3,2,3,2,1)\}$$

___________ **g.** The fixed points $\mathbf{x}$ of a matrix transformation $T(\mathbf{x}) = A\mathbf{x}$ are eigenvectors with eigenvalue 1.

#1.[18]	
#2.[18]	
#3.[10]	
#4.[12]	
#5.[10]	
#6.[12]	
#7.[21]	
Total	