CLARK

Name: ____________________________

Mailbox number:____________________

Math 130 Linear Algebra

Second Test

November 2013

You may use a calculator and a sheet of notes. Leave your answers as expressions such as $e^2\sqrt{\dfrac{\sin^2(\pi/6)}{1+\ln 10}}$ if you like. Show all your work for credit. Points for each problem are in square brackets.

1. [18] Give examples of transformations with the following properties. You don't have to prove that they have the properties. Just specify the transformations.

a. [5] A linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ whose rank is 0 and nullity is 2.

b. [5] A linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ whose rank is 1 and nullity is 1.

c. [8] Two different linear transformations $\mathbf{R}^2 \to \mathbf{R}^2$ both with rank 2 and nullity 0.

2. [20] The set of five vectors

$$S = \{(1, -3, -2), (1, 2, 2), (3, 1, 2), (0, 5, 4), (2, -1, 1)\}$$

spans $\mathbf{R}^3$. Find a subset of S which is a basis for $\mathbf{R}^3$. Show your work.

3. [28; 4 points each] True/false. For each sentence write the whole word "true" or the whole word "false". If it's not clear whether it should be considered true or false, you may explain in a sentence if you like.

___________ **a.** If $T : V \to W$ is a linear transformation, and if the vectors $T(\mathbf{v}_1), \ldots, T(\mathbf{v}_k)$ are linearly independent in W, then the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_k$ are linearly independent in V.

___________ **b.** If a linear transformation $T : V \to W$ goes between two vector spaces V and W of the same dimension, then it's an isomorphism.

___________ **c.** If the rank of a linear transformation $T : \mathbf{R}^n \to \mathbf{R}^m$ is m, then T is surjective (that is to say, T is onto).

___________ **d.** Given two $n \times n$ matrices A and B, if $(A + B)(A - B) = A^2 - B^2$, then $AB = BA$.

___________ **e.** Given two $n \times n$ matrices A and B, if both A and B are invertible, then so is $A + B$.

___________ **f.** The vector space $M_{2\times 6}$ of 2×6 matrices is isomorphic to the vector space $M_{3\times 4}$ of 3×4 matrices.

___________ **g.** If a set S spans a vector space V , then every every vector in V can be written as a linear combination of vectors from S in only one way.

4. [20] Consider the linear transformation $T : \mathbf{R}^5 \to \mathbf{R}^3$ described by the matrix A, that is, $T(\mathbf{x})$ is found by evaluating A times the column matrix $\mathbf{x}$, where

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

a. [7] Determine a basis for the kernel of T. (This should be a set of vectors in $\mathbf{R}^5$.)

b. [3] What is the nullity of T?

c. [7] Determine a basis for the image of T. (This should be a set of vectors in $\mathbf{R}^3$.)

d. [3] What is the rank of T?

5. [15] If A is a 5×3 matrix, prove that the rows of A are linearly dependent. (There are several ways that you can approach this. Be sure to write a clear and complete explanation.)

#1.[18]	
#2.[20]	
#3.[28]	
#4.[20]	
#5.[15]	
Total	